scikit-fmm Documentation

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`scikit-fmm` is a python extension module which implements the fast marching method.

The fast marching method is used to model the evolution of boundaries and interfaces in a variety of application areas.

More specifically, the fast marching method is a numerical technique for finding approximate solutions to boundary value problems of the Eikonal equation,

$$F(x)|\nabla T(x)| = 1$$

Typically, such a problem describes the evolution of a closed curve as a function of time T with speed $F(x) > 0$ in the normal direction at a point x on the curve. The speed function is specified, and the time at which the contour crosses a point x is obtained by solving the equation. The initial location of the boundary is defined by the zero contour (or zero level-set) of a scalar function.

In this document the scalar function containing the initial interface location is referred to as phi. The scalar function phi can be thought to exist in a dimension higher than the boundary of interest and only the zero contour of the function is physically meaningful. The boundary grows outward in the local normal direction at a speed given by $F(x)$.

`scikit-fmm` is a simple module which provides the functions: *`distance()`*, *`travel_time()`* and *`extension_velocities()`*. The import name of `scikit-fmm` is *`skfmm`*.

CHAPTER 1

Examples

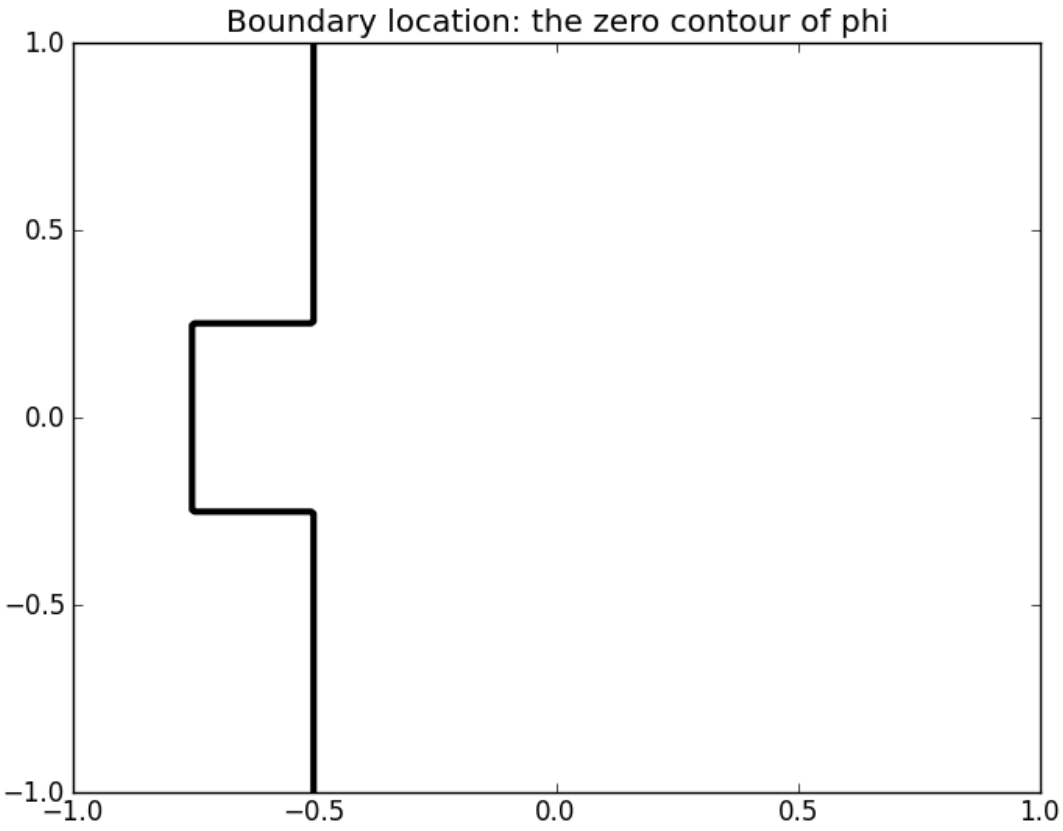
First, a simple example:

```
>>> import skfmm
>>> import numpy as np
>>> phi = np.ones((3, 3))
>>> phi[1, 1] = -1
>>> skfmm.distance(phi)
array([[ 1.20710678,  0.5       ,  1.20710678],
       [ 0.5       , -0.35355339,  0.5       ],
       [ 1.20710678,  0.5       ,  1.20710678]])
```

Here the zero contour of phi is around the (1, 1) point. The return value of *distance()* gives the signed distance from zero contour. No grid spacing is given, so it is taken as 1. To specify a spacing use the optional dx argument:

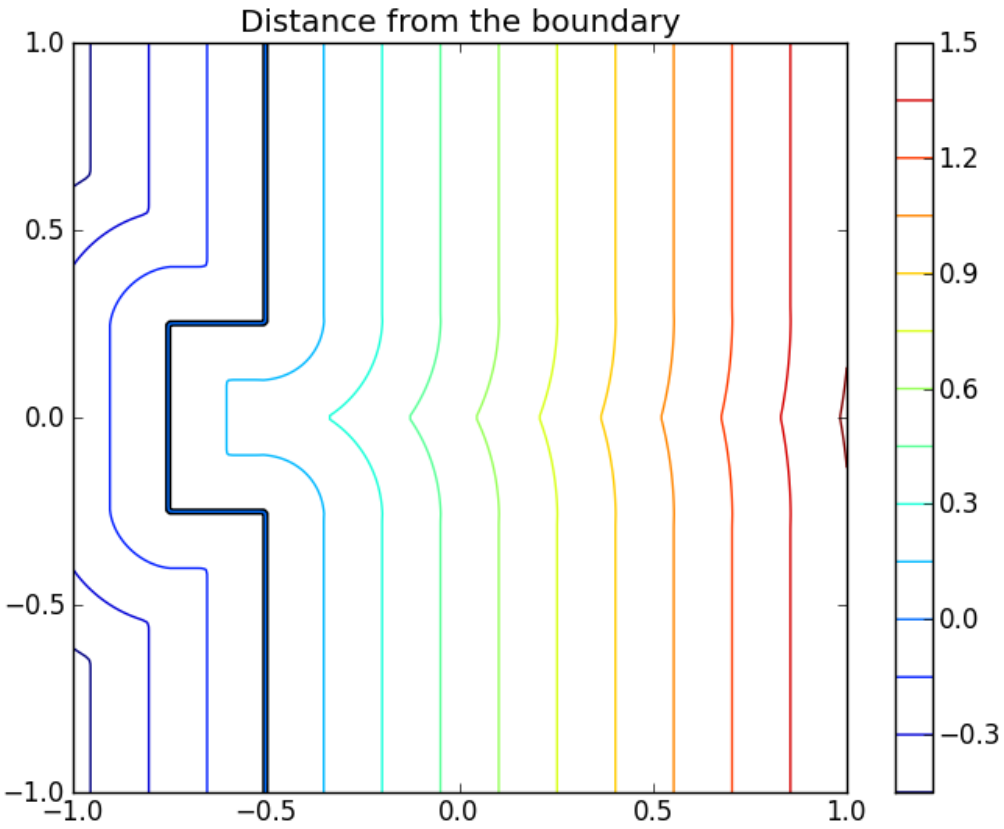
```
>>> skfmm.distance(phi, dx=0.25)
array([[ 0.3017767 ,  0.125     ,  0.3017767 ],
       [ 0.125     , -0.08838835,  0.125     ],
       [ 0.3017767 ,  0.125     ,  0.3017767 ]])
```

A more detailed example:



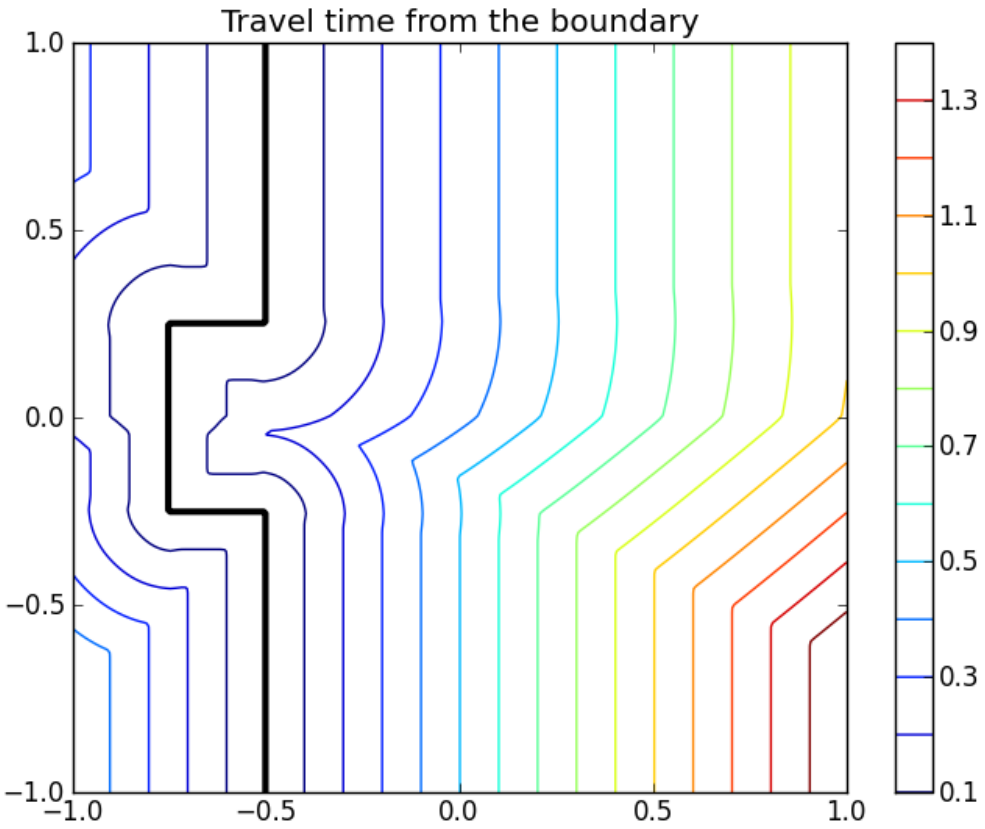
The boundary is specified as the zero contour of a scalar function phi:

```
>>> import numpy as np
>>> import pylab as pl
>>> X, Y = np.meshgrid(np.linspace(-1,1,200), np.linspace(-1,1,200))
>>> phi = -1 * np.ones_like(X)
>>> phi[X > -0.5] = 1
>>> phi[np.logical_and(np.abs(Y) < 0.25, X > -0.75)] = 1
>>> d = skfmm.distance(phi, dx=1e-2)
```



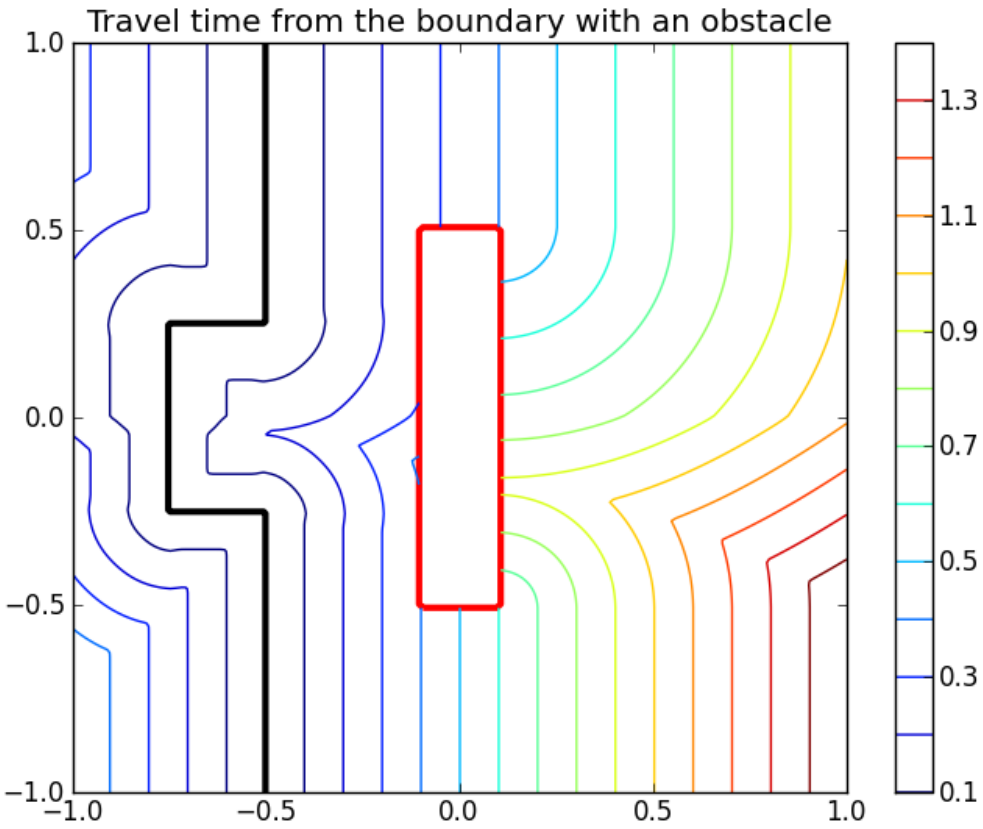
`scikit-fmm` can also calculate travel times from an interface given an array containing the interface propogation speed at each point. Using the same initial interface position as above we set the interface propagation speed to be 1.5 times greater in the upper half of the domain.

```
>>> speed = np.ones_like(X)
>>> speed[Y > 0] = 1.5
>>> t = skfmm.travel_time(phi, speed, dx=1e-2)
```



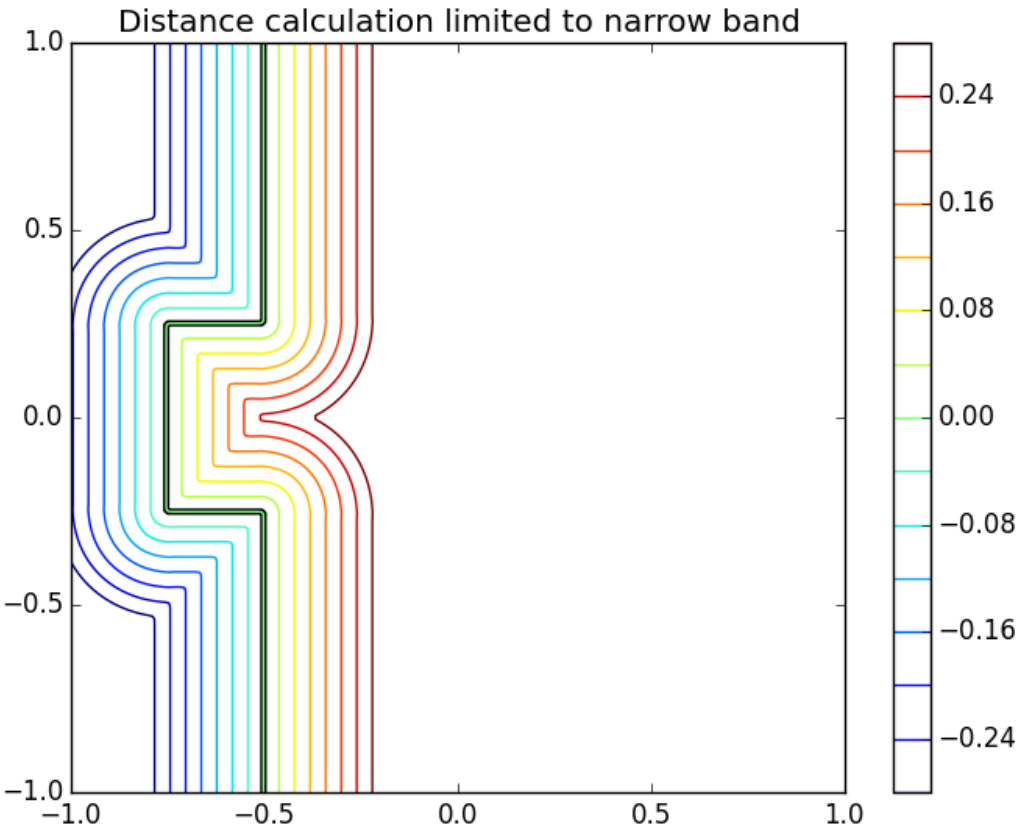
Consider an obstacle within which the speed is vanishing. In principle this may lead to singularities (division by zero) in the update algorithm. Therefore, both *travel_time()* and *distance()* support masked arrays for input. This allows an obstacle to be introduced. (Note that if the speed in a cell is less than machine precision, a cell is masked internally to prevent division by 0.)

```
>>> mask = np.logical_and(abs(X) < 0.1, abs(Y) < 0.5)
>>> phi  = np.ma.MaskedArray(phi, mask)
>>> t    = skfmm.travel_time(phi, speed, dx=1e-2)
```



The distance function, travel time or extension velocities can be limited to with in a narrow band around the zero contour by specifying the *narrow* keyword.

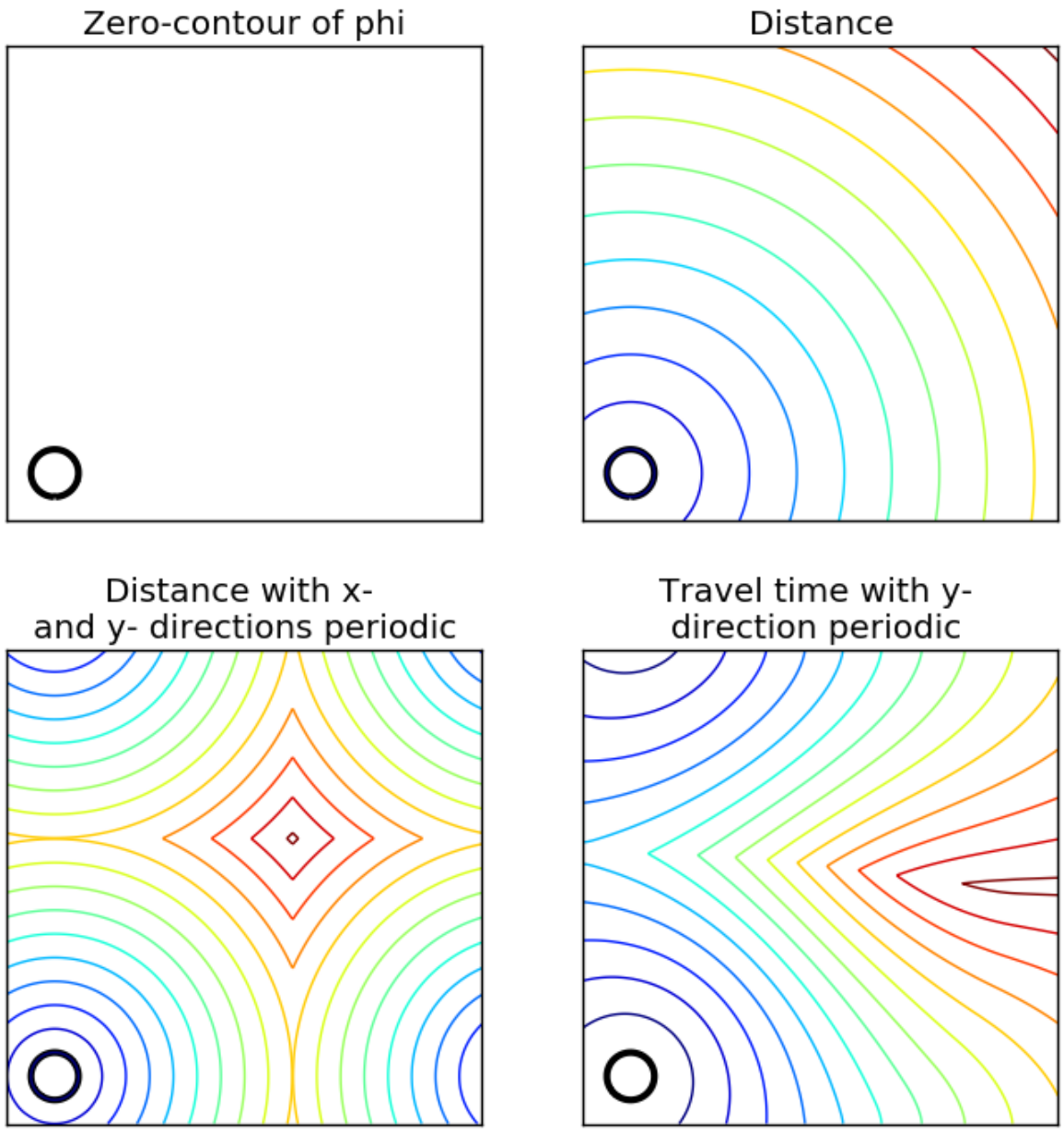
```
>>> phi = -1 * np.ones_like(X)
>>> phi[X > -0.5] = 1
>>> phi[np.logical_and(np.abs(Y) < 0.25, X > -0.75)] = 1
>>> d = skfmm.distance(phi, dx=1e-2, narrow=0.3)
```



The full example is in examples/2d_example.py. Example Listing

An example of using periodic boundary conditions.

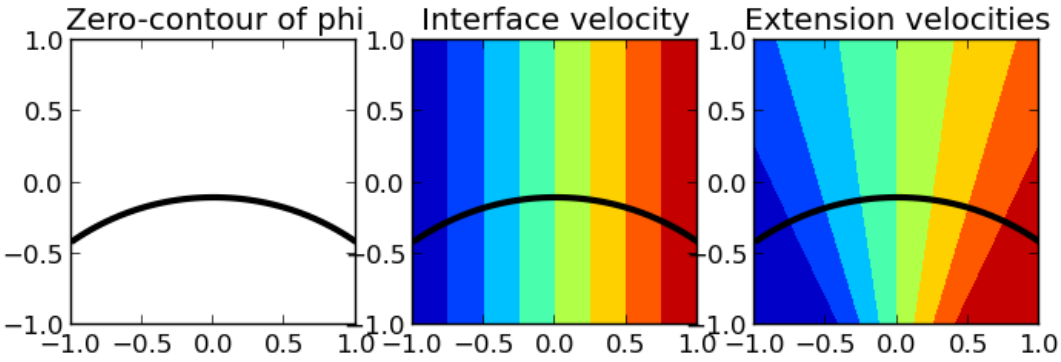
```
>>> X, Y = np.meshgrid(np.linspace(-1,1,501), np.linspace(-1,1,501))
>>> phi = (X+0.8)**2+(Y+0.8)**2 - 0.01
>>> speed = 1+X**2+Y**2
>>> skfmm.distance(phi, dx=2.0/500)
>>> skfmm.distance(phi, dx=2.0/500, periodic=True)
>>> skfmm.travel_time(phi, speed, dx=2.0/500, periodic=(1,0))
```



The full example is in examples/boundaryconditions_example.py Example Listing

An example of using `scikit-fmm` to compute extension velocities.

```
>>> N      = 150
>>> X, Y   = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r      = 1.75
>>> dx     = 2.0 / (N - 1)
>>> phi    = (X) ** 2 + (Y+1.85) ** 2 - r ** 2
>>> speed = X + 1.25
>>> d, f_ext = extension_velocities(phi, speed, dx)
```



The full example is in examples/extension_velocities_example.py. Example Listing

Example Listing

2d_example.py

```
import numpy as np
import pylab as plt
import skfmm

X, Y = np.meshgrid(np.linspace(-1,1,201), np.linspace(-1,1,201))
phi = -1*np.ones_like(X)

phi[X>-0.5] = 1
phi[np.logical_and(np.abs(Y)<0.25, X>-0.75)] = 1
plt.contour(X, Y, phi,[0], linewidths=(3), colors='black')
plt.title('Boundary location: the zero contour of phi')
plt.savefig('2d_phi.png')
plt.show()

d = skfmm.distance(phi, dx=1e-2)
plt.title('Distance from the boundary')
plt.contour(X, Y, phi,[0], linewidths=(3), colors='black')
```

```
plt.contour(X, Y, d, 15)
plt.colorbar()
plt.savefig('2d_phi_distance.png')
plt.show()

speed = np.ones_like(X)
speed[Y>0] = 1.5
t = skfmm.travel_time(phi, speed, dx=1e-2)

plt.title('Travel time from the boundary')
plt.contour(X, Y, phi,[0], linewidths=(3), colors='black')
plt.contour(X, Y, t, 15)
plt.colorbar()
plt.savefig('2d_phi_travel_time.png')
plt.show()

mask = np.logical_and(abs(X)<0.1, abs(Y)<0.5)
phi  = np.ma.MaskedArray(phi, mask)
t    = skfmm.travel_time(phi, speed, dx=1e-2)
plt.title('Travel time from the boundary with an obstacle')
plt.contour(X, Y, phi, [0], linewidths=(3), colors='black')
plt.contour(X, Y, phi.mask, [0], linewidths=(3), colors='red')
plt.contour(X, Y, t, 15)
plt.colorbar()
plt.savefig('2d_phi_travel_time_mask.png')
plt.show()

phi = -1 * np.ones_like(X)
phi[X > -0.5] = 1
phi[np.logical_and(np.abs(Y) < 0.25, X > -0.75)] = 1
d = skfmm.distance(phi, dx=1e-2, narrow=0.3)
plt.title('Distance calculation limited to narrow band')
plt.contour(X, Y, phi, [0], linewidths=(3), colors='black')
plt.contour(X, Y, d, 15)
plt.colorbar()
plt.savefig('2d_phi_distance_narrow.png')
plt.show()
```

boundaryconditions_example.py

```
import numpy as np
import pylab as plt
import skfmm

X, Y = np.meshgrid(np.linspace(-1,1,501), np.linspace(-1,1,501))
phi = (X+0.8)**2+(Y+0.8)**2 - 0.01
speed = 1+X**2+Y**2

plt.subplot(221)
plt.title("Zero-contour of phi")
plt.contour(X, Y, phi, [0], colors='black', linewidths=(3))
plt.gca().set_aspect(1)
plt.xticks([]); plt.yticks([])

plt.subplot(222)
plt.title("Distance")
```

```
plt.contour(X, Y, phi, [0], colors='black', linewidths=(3))
plt.contour(X, Y, skfmm.distance(phi, dx=2.0/500), 15)
plt.gca().set_aspect(1)
plt.xticks([]); plt.yticks([])

plt.subplot(223)
plt.title("Distance with x- \nand y- directions periodic")
plt.contour(X, Y, phi, [0], colors='black', linewidths=(3))
plt.contour(X, Y, skfmm.distance(phi, dx=2.0/500, periodic=True), 15)
plt.gca().set_aspect(1)
plt.xticks([]); plt.yticks([])

plt.subplot(224)
plt.title("Travel time with y- \ndirection periodic ")
plt.contour(X, Y, phi, [0], colors='black', linewidths=(3))
plt.contour(X, Y, skfmm.travel_time(phi, speed, dx=2.0/500, periodic=(1,0)), 15)
plt.gca().set_aspect(1)
plt.xticks([]); plt.yticks([])

plt.show()
```

extension_velocities_example.py

```
import numpy as np
import pylab as plt
from skfmm import extension_velocities

N     = 150
X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
r     = 1.75
dx    = 2.0 / (N - 1)
phi   = (X) ** 2 + (Y+1.85) ** 2 - r ** 2
speed = X+1.25
d, f_ext = extension_velocities(phi, speed, dx)


plt.subplot(131)
plt.title("Zero-contour of phi")
plt.contour(X, Y, phi,[0], colors='black', linewidths=(3))
plt.gca().set_aspect(1)

plt.subplot(132)
plt.title("Interface velocity")
plt.contour(X, Y, phi,[0], colors='black', linewidths=(3))
plt.contourf(X, Y, speed)
plt.gca().set_aspect(1)

plt.subplot(133)
plt.title("Extension velocities")
plt.contour(X, Y, phi,[0], colors='black', linewidths=(3))
plt.contourf(X, Y, f_ext)
plt.gca().set_aspect(1)

plt.show()
```

Doctests

skfmm.**testing**()

These tests are gathered from FiPy, PyLSMLIB and original Scikit-fmm tests.

1D Test

```
>>> import numpy as np
```

```
>>> print(np.allclose(distance((-1., -1., -1., -1., 1., 1., 1., 1.), dx=.5),
...                   (-1.75, -1.25, -.75, -0.25, 0.25, 0.75, 1.25, 1.75)))
True
```

Small dimensions.

```
>>> dx = 1e-10
>>> print(np.allclose(distance((-1., -1., -1., -1., 1., 1., 1., 1.), dx=dx),
...                   np.arange(8) * dx - 3.5 * dx))
True
```

Bug Fix

Test case for a bug in the upwind finite difference scheme for negative phi. When computing finite differences we want to preferentially use information from the frozen neighbors that are closest to the zero contour in each dimension. This means that we must compare absolute distances when checking neighbors in the negative phi direction.

The error can result in incorrect values of the updated signed distance function for regions close to the minimum contour of the level set function, i.e. in the middle of holes.

To test we use a square matrix for the initial phi field that is equal to -1 on the main diagonal and on the three diagonals above and below this. The matrix is set to 1 everywhere else. The bug results in errors in positions (1,1), (2,2), (3,3), (6,6), (7,7) and (8,8) along the main diagonal.

This error occurs for first- and second-order updates. For simplicity, we choose to only test the first-order update.

```
>>> phi = np.ones((10, 10))
>>> i,j = np.indices(phi.shape)
>>> phi[i==j-3] = -1
>>> phi[i==j-2] = -1
>>> phi[i==j-1] = -1
>>> phi[i==j] = -1
>>> phi[i==j+1] = -1
>>> phi[i==j+2] = -1
>>> phi[i==j+3] = -1
>>> phi = distance(phi, order=1)
>>> print(np.allclose(phi[1, 1], -2.70464, atol=1e-4))
True
>>> print(np.allclose(phi[2, 2], -2.50873, atol=1e-4))
True
>>> print(np.allclose(phi[3, 3], -2.47487, atol=1e-4))
True
>>> print(np.allclose(phi[6, 6], -2.47487, atol=1e-4))
True
>>> print(np.allclose(phi[7, 7], -2.50873, atol=1e-4))
True
>>> print(np.allclose(phi[8, 8], -2.70464, atol=1e-4))
True
```

Bug Fix

A 2D test case to test trial values for a pathological case.

```
>>> dx = 1.
>>> dy = 2.
>>> vbl = -dx * dy / np.sqrt(dx**2 + dy**2) / 2.
>>> vbr = dx / 2
>>> vml = dy / 2.
>>> crossProd = dx * dy
>>> dsq = dx**2 + dy**2
>>> top = vbr * dx**2 + vml * dy**2
>>> sqrt = crossProd**2 *(dsq - (vbr - vml)**2)
>>> sqrt = np.sqrt(max(sqrt, 0))
>>> vmr = (top + sqrt) / dsq
>>> print(np.allclose(distance(((-1., 1., -1.), (1., 1., 1.)), dx=(dx, dy), order=1),
...                   ((vbl, vml, vbl), (vbr, vmr, vbr))))
True
```

Test Extension Field Calculation

```
>>> tmp = 1 / np.sqrt(2)
>>> phi = np.array([[-1., 1.], [1., 1.]])
>>> phi, ext =  extension_velocities(phi,
...                                  [[-1, .5], [2., -1.]],
...                                  ext_mask=phi < 0,
...                                  dx=1., order=1)
>>> print(np.allclose(phi, ((-tmp / 2, 0.5), (0.5, 0.5 + tmp))))
True
>>> print(np.allclose(ext, [[1.25, .5], [2., 1.25]]))
True
```

```
>>> phi = np.array(((-1., 1., 1.), (1., 1., 1.), (1., 1., 1.)))
>>> phi, ext = extension_velocities(phi,
...                                 ((-1., 2., -1.),
...                                  (.5, -1., -1.),
...                                  (-1., -1., -1.)),
...                                 ext_mask=phi < 0,
...                                 order=1)
>>> v1 = 0.5 + tmp
>>> v2 = 1.5
>>> tmp1 = (v1 + v2) / 2 + np.sqrt(2. - (v1 - v2)**2) / 2
>>> tmp2 = tmp1 + 1 / np.sqrt(2)
>>> print(np.allclose(phi, ((-tmp / 2, 0.5, 1.5),
...                         (0.5, 0.5 + tmp, tmp1),
...                         (1.5, tmp1, tmp2))))
True
>>> print(np.allclose(ext, ((1.25, 2., 2.),
...                         (.5, 1.25, 1.5456),
...                         (.5, 0.9544, 1.25)),
...                   rtol = 1e-4))
True
```

Bug Fix

Test case for a bug that occurs when initializing the distance variable at the interface. Currently it is assumed that adjacent cells that are opposite sign neighbors have perpendicular normal vectors. In fact the two closest cells could have opposite normals.

```
>>> print(np.allclose(distance((-1., 1., -1.)), (-0.5, 0.5, -0.5)))
True
```

Testing second order. This example failed with Scikit-fmm.

```
>>> phi = ((-1., -1., 1., 1.),
...        (-1., -1., 1., 1.),
...        (1., 1., 1., 1.),
...        (1., 1., 1., 1.))
>>> answer = ((-1.30473785, -0.5, 0.5, 1.49923009),
...           (-0.5, -0.35355339, 0.5, 1.45118446),
...           (0.5, 0.5, 0.97140452, 1.76215286),
...           (1.49923009, 1.45118446, 1.76215286, 2.33721352))
>>> print(np.allclose(distance(phi),
...                   answer,
...                   rtol=1e-9))
True
```

A test for a bug in both LSMLIB and Scikit-fmm

The following test gives different results depending on whether LSMLIB or Scikit-fmm is used. This issue occurs when calculating second order accurate distance functions. When a value becomes "known" after previously being a "trial" value it updates its neighbors' values. In a second order scheme the neighbors one step away also need to be updated (if the cell between the new "known" cell and the cell required for second order accuracy also happens to be "known"), but are not updated in either package. By luck (due to trial values having the same value), the values calculated in Scikit-fmm for the following example are correct although an example that didn't work for Scikit-fmm could also be constructed.

```
>>> phi = distance([[-1, -1, -1, -1],
...                 [ 1,  1, -1, -1],
...                 [ 1,  1, -1, -1],
...                 [ 1,  1, -1, -1]], order=2)
>>> phi = distance(phi, order=2)
```

The following values come form Scikit-fmm.

```
>>> answer = [[-0.5,         -0.58578644, -1.08578644, -1.85136395],
...           [ 0.5,          0.29289322, -0.58578644, -1.54389939],
...           [ 1.30473785,  0.5,         -0.5,         -1.5       ],
...           [ 1.49547948,  0.5,         -0.5,         -1.5       ]]
```

The 3rd and 7th element are different for LSMLIB. This is because the 15th element is not "known" when the "trial" value for the 7th element is calculated. Scikit-fmm calculates the values in a slightly different order so gets a seemingly better answer, but this is just chance.

```
>>> print(np.allclose(phi, answer, rtol=1e-9))
True
```

The following tests for the same issue but is a better test case guaranteed to fail.

```
>>> phi = np.array([[-1,  1,  1,  1,  1, -1],
...                 [-1, -1, -1, -1, -1, -1],
...                 [-1, -1, -1, -1, -1, -1]])
```

```
>>> phi = distance(phi)
>>> print(phi[2, 2] == phi[2, 3])
True
```

```
>>> phi = distance(phi)
>>> print(phi[2, 2] == phi[2, 3])
True
```

Circle Example

Solve the level set equation in two dimensions for a circle.

The 2D level set equation can be written,

$$|\nabla\phi| = 1$$

and the boundary condition for a circle is given by, $\phi = 0$ at $(x - L/2)^2 + (y - L/2)^2 = (L/4)^2$.

The solution to this problem will be demonstrated in the following script. Firstly, setup the parameters.

```
>>> def mesh(nx=1, ny=1, dx=1., dy=1.):
...     y, x = np.mgrid[0:nx,0:ny]
...     x = x * dx + dx / 2
...     y = y * dy + dy / 2
...     return x, y
```

```
>>> dx = 1.
>>> N = 11
>>> L = N * dx
>>> x, y = mesh(nx=N, ny=N, dx=dx, dy=dx)
>>> phi = -np.ones(N * N, 'd')
>>> phi[(x.flatten() - L / 2.)**2 + (y.flatten() - L / 2.)**2 < (L / 4.)**2] = 1.
>>> phi = np.reshape(phi, (N, N))
>>> phi = distance(phi, dx=dx, order=1).flatten()
```

```
>>> dX = dx / 2.
>>> m1 = dX * dX / np.sqrt(dX**2 + dX**2)
>>> def evalCell(phix, phiy, dx):
...     aa = dx**2 + dx**2
...     bb = -2 * ( phix * dx**2 + phiy * dx**2)
...     cc = dx**2 * phix**2 + dx**2 * phiy**2 - dx**2 * dx**2
...     sqr = np.sqrt(bb**2 - 4. * aa * cc)
...     return ((-bb - sqr) / 2. / aa,  (-bb + sqr) / 2. / aa)
>>> v1 = evalCell(-dX, -m1, dx)[0]
>>> v2 = evalCell(-m1, -dX, dx)[0]
>>> v3 = evalCell(m1,  m1,  dx)[1]
>>> v4 = evalCell(v3, dX, dx)[1]
>>> v5 = evalCell(dX, v3, dx)[1]
>>> MASK = -1000.
>>> trialValues = np.array((
...     MASK,  MASK, MASK, MASK, MASK, MASK, MASK, MASK, MASK, MASK, MASK,
...     MASK,  MASK, MASK, MASK,-3*dX,-3*dX,-3*dX, MASK, MASK, MASK, MASK,
...     MASK,  MASK, MASK,   v1,  -dX,  -dX,  -dX,   v1, MASK, MASK, MASK,
...     MASK,  MASK,   v2,  -m1,   m1,   dX,   m1,  -m1,   v2, MASK, MASK,
...     MASK, -dX*3,  -dX,   m1,   v3,   v4,   v3,   m1,  -dX,-dX*3, MASK,
...     MASK, -dX*3,  -dX,   dX,   v5, MASK,   v5,   dX,  -dX,-dX*3, MASK,
...     MASK, -dX*3,  -dX,   m1,   v3,   v4,   v3,   m1,  -dX,-dX*3, MASK,
...     MASK,  MASK,   v2,  -m1,   m1,   dX,   m1,  -m1,   v2, MASK, MASK,
...     MASK,  MASK, MASK,   v1,  -dX,  -dX,  -dX,   v1, MASK, MASK, MASK,
...     MASK,  MASK, MASK, MASK,-3*dX,-3*dX,-3*dX, MASK, MASK, MASK, MASK,
...     MASK,  MASK, MASK, MASK, MASK, MASK, MASK, MASK, MASK, MASK, MASK), 'd')
```

```
>>> phi[trialValues == MASK] = MASK
>>> print(np.allclose(phi, trialValues))
True
```

Square Example

Here we solve the level set equation in two dimensions for a square. The equation is given by:

$$|\nabla\phi| = 1$$

$$\phi = 0 \qquad \text{at} \qquad \begin{cases} x = (L/3, 2L/3) & \text{for } L/3 \le y \le 2L/3 \\ y = (L/3, 2L/3) & \text{for } L/3 \le x \le 2L/3 \end{cases}$$

```
>>> dx = 0.5
>>> dy = 2.
>>> nx = 5
>>> ny = 5
>>> Lx = nx * dx
>>> Ly = ny * dy
```

```
>>> x, y = mesh(nx=nx, ny=ny, dx=dx, dy=dy)
>>> x = x.flatten()
>>> y = y.flatten()
>>> phi = -np.ones(nx * ny, 'd')
>>> phi[((Lx / 3. < x) & (x < 2. * Lx / 3.)) & ((Ly / 3. < y) & (y < 2. * Ly / 3))] = 1.
>>> phi = np.reshape(phi, (nx, ny))
>>> phi = distance(phi, dx=(dy, dx), order=1).flatten()
```

```
>>> def evalCell(phix, phiy, dx, dy):
...     aa = dy**2 + dx**2
...     bb = -2 * ( phix * dy**2 + phiy * dx**2)
...     cc = dy**2 * phix**2 + dx**2 * phiy**2 - dx**2 * dy**2
...     sqr = np.sqrt(bb**2 - 4. * aa * cc)
...     return ((-bb - sqr) / 2. / aa,  (-bb + sqr) / 2. / aa)
>>> val = evalCell(-dy / 2., -dx / 2., dx, dy)[0]
>>> v1 = evalCell(val, -3. * dx / 2., dx, dy)[0]
>>> v2 = evalCell(-3. * dy / 2., val, dx, dy)[0]
>>> v3 = evalCell(v2, v1, dx, dy)[0]
>>> v4 = dx * dy / np.sqrt(dx**2 + dy**2) / 2
>>> arr = np.array((
...     v3          , v2      , -3. * dy / 2.   , v2      , v3,
...     v1          , val     , -dy / 2.        , val     , v1          ,
...     -3. * dx / 2., -dx / 2., v4             , -dx / 2., -3. * dx / 2.,
...     v1          , val     , -dy / 2.        , val     , v1          ,
...     v3          , v2      , -3. * dy / 2.   , v2      , v3          ))
>>> print(np.allclose(arr, phi))
True
```

Assertion Errors

```
>>> distance([[-1, 1],[1, 1]], dx=(1, 2, 3))
Traceback (most recent call last):
  ...
ValueError: dx must be of length len(phi.shape)
>>> extension_velocities([[-1, 1],[1, 1]], speed=[1, 1])
Traceback (most recent call last):
  ...
ValueError: phi and speed must have the same shape
```

Test for 1D equality between 'distance' and 'travel_time'

```
>>> phi = np.arange(-5, 5) + 0.499
>>> d = distance(phi)
>>> t = travel_time(phi, speed=np.ones_like(phi))
>>> np.testing.assert_allclose(t, np.abs(d))
```

Tests taken from FiPy

```
>>> phi = np.array(((-1, -1, 1, 1),
...                 (-1, -1, 1, 1),
...                 (1, 1, 1, 1),
...                 (1, 1, 1, 1)))
>>> o1 = distance(phi, order=1, self_test=True)
>>> dw_o1 =   [[-1.20710678, -0.5,         0.5,         1.5],
...            [-0.5,        -0.35355339,  0.5,         1.5],
...            [ 0.5,         0.5,         1.20710678,  2.04532893],
...            [ 1.5,         1.5,         2.04532893,  2.75243571]]
>>> np.testing.assert_allclose(o1, dw_o1)
```

```
>>> phi = np.array(((-1, -1, 1, 1),
...                 (-1, -1, 1, 1),
...                 (1, 1, 1, 1),
...                 (1, 1, 1, 1)))
>>> o1 = travel_time(phi, np.ones_like(phi), order=1, self_test=True)
>>> dw_o1 =   [[-1.20710678, -0.5,         0.5,         1.5],
...            [-0.5,        -0.35355339,  0.5,         1.5],
...            [ 0.5,         0.5,         1.20710678,  2.04532893],
...            [ 1.5,         1.5,         2.04532893,  2.75243571]]
>>> np.testing.assert_allclose(o1, np.abs(dw_o1))
```

```
>>> phi = np.array(((-1, -1, 1, 1),
...                 (-1, -1, 1, 1),
...                 (1, 1, 1, 1),
...                 (1, 1, 1, 1)))
>>> o2 = distance(phi, self_test=True)
>>> dw_o2 = [[-1.30473785,  -0.5,          0.5,         1.49923009],
...          [-0.5,        -0.35355339,    0.5,         1.45118446],
...          [ 0.5,         0.5,         0.97140452,  1.76215286],
...          [ 1.49923009,  1.45118446,  1.76215286,  2.33721352]]
```

```
>>> np.testing.assert_allclose(o2, dw_o2)
>>> phi = np.array(((-1, -1, 1, 1),
...                 (-1, -1, 1, 1),
...                 (1, 1, 1, 1),
...                 (1, 1, 1, 1)))
>>> o2 = travel_time(phi, np.ones_like(phi), self_test=True)
>>> dw_o2 = [[-1.30473785,  -0.5,          0.5,         1.49923009],
...          [-0.5,        -0.35355339,    0.5,         1.45118446],
...          [ 0.5,         0.5,         0.97140452,  1.76215286],
...          [ 1.49923009,  1.45118446,  1.76215286,  2.33721352]]
>>> np.testing.assert_allclose(o2, np.abs(dw_o2))
```

```
>>> distance([-1,1], order=0)
Traceback (most recent call last):
  ...
ValueError: order must be 1 or 2
```

```
>>> distance([-1,1], order=3)
Traceback (most recent call last):
  ...
ValueError: order must be 1 or 2
```

Extension velocity tests

Test 1d extension constant.

```
>>> phi =   [-1,-1,-1,1,1,1]
>>> speed = [1,1,1,1,1,1]
>>> d, f_ext  = extension_velocities(phi, speed, self_test=True)
>>> np.testing.assert_allclose(speed, f_ext)
```

Test the 1D extension block.

```
>>> phi =   np.ones(10)
>>> phi[0] =- 1
>>> speed = np.ones(10)
>>> speed[0:3] = 5
>>> d, f_ext  = extension_velocities(phi, speed, self_test=True)
>>> np.testing.assert_allclose(f_ext, 5)
```

Test that a uniform speed value is preserved.

```
>>> N     = 50
>>> X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r     = 0.25
>>> dx    = 2.0 / (N - 1)
>>> phi   = (X) ** 2 + (Y) ** 2 - r ** 2
>>> speed = np.ones_like(phi)
>>> d, f_ext = extension_velocities(phi, speed, dx, self_test=True)
>>> np.testing.assert_allclose(f_ext, 1.0)
```

Constant value march-out test

```
>>> speed[abs(Y)<0.3] = 10.0
>>> d, f_ext = extension_velocities(phi, speed, dx, self_test=True)
>>> np.testing.assert_allclose(f_ext, 10.0)
```

Test distance from extension

```
>>> speed = np.ones_like(phi)
>>> d, f_ext = extension_velocities(phi, speed, dx, self_test=True)
>>> d2 = distance(phi, dx, self_test=True)
>>> np.testing.assert_allclose(d, d2)
```

Test for extension velocity bug

```
>>> N     = 150
>>> X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r     = 0.5
>>> dx    = 2.0 / (N - 1)
>>> phi   = (X) ** 2 + (Y) ** 2 - r ** 2
>>> speed = np.ones_like(phi)
>>> speed[X>0.25] = 3.0
>>> d2, f_ext = extension_velocities(phi, speed, dx)
```

```
>>> assert (f_ext <= 3.0000001).all()
>>> assert (f_ext >= 1).all()
```

```
>>> np.testing.assert_almost_equal(f_ext[137, 95], 1, 3)
>>> np.testing.assert_almost_equal(f_ext[103, 78], 1, 2)
>>> np.testing.assert_almost_equal(f_ext[72, 100], 3, 3)
>>> np.testing.assert_almost_equal(f_ext[72, 86], 3, 3)
>>> np.testing.assert_almost_equal(f_ext[110, 121], 3, 3)
```

Simple two point tests

```
>>> np.testing.assert_array_equal(distance([-1, 1]),
...                                        [-0.5, 0.5])
>>> np.testing.assert_allclose(distance([-1, -1, -1, 1, 1, 1]),
...                                    [-2.5, -1.5, -0.5, 0.5, 1.5, 2.5])
>>> np.testing.assert_allclose(distance([1, 1, 1, -1, -1, -1]),
...                                    [2.5, 1.5, 0.5, -0.5, -1.5, -2.5])
```

Three point test case

```
>>> np.testing.assert_array_equal(distance([-1, 0, 1]),           [-1, 0, 1])
>>> np.testing.assert_array_equal(distance([-1, 0, 1], dx=[2]),   [-2, 0, 2])
>>> np.testing.assert_array_equal(distance([-1, 0, 1], dx=2),     [-2, 0, 2])
>>> np.testing.assert_array_equal(distance([-1, 0, 1], dx=2.0),   [-2, 0, 2])
>>> np.testing.assert_array_equal(travel_time([1, 0, -1], [1, 1, 1]),
...                                           [1, 0, 1])
>>> np.testing.assert_array_equal(travel_time([-1, 0, 1], [1, 1, 1]),
...                                           [1, 0, 1])
>>> np.testing.assert_array_equal(travel_time([1, 0, -1], [1, 1, 1], dx=2),
...                                           [2, 0, 2])
>>> np.testing.assert_array_equal(travel_time([1, 0, -1], [1, 1, 1], dx=[2]),
...                                           [2, 0, 2])
>>> np.testing.assert_array_equal(travel_time([1, 0, -1], [1, 1, 1], dx=2.0),
...                                           [2, 0, 2])
```

Travel time tests 1

```
>>> np.testing.assert_allclose(travel_time([0, 1, 1, 1, 1], [2, 2, 2, 2, 2]),
...                                        [0, 0.5, 1.0, 1.5, 2.0])
>>> np.testing.assert_array_equal(travel_time([1, 0, -1], [2, 2, 2]),
...                                           [0.5, 0, 0.5])
```

Travel time tests 2

```
>>> phi   = [1, 1, 1, -1, -1, -1]
>>> t     = travel_time(phi, np.ones_like(phi))
>>> exact = [2.5, 1.5, 0.5, 0.5, 1.5, 2.5]
>>> np.testing.assert_allclose(t, exact)
```

Travel time tests 3

```
>>> phi   = [-1, -1, -1, 1, 1, 1]
>>> t     = travel_time(phi, np.ones_like(phi))
>>> exact = [2.5, 1.5, 0.5, 0.5, 1.5, 2.5]
>>> np.testing.assert_allclose(t, exact)
```

Corner case

```
>>> np.testing.assert_array_equal(distance([0, 0]), [0, 0])
>>> np.testing.assert_array_equal(travel_time([0, 0], [1, 1]), [0, 0])
```

Test zero

```
>>> distance([1, 0, 1, 1], 0)
Traceback (most recent call last):
  ...
ValueError: dx must be greater than zero
```

Test dx shape

```
>>> distance([0, 0, 1, 0, 0], [0, 0, 1, 0, 0])
Traceback (most recent call last):
  ...
ValueError: dx must be of length len(phi.shape)
```

Test for small speeds

Test catching speeds which are too small. Speeds less than the machine epsilon are masked off to avoid an overflow.

```
>>> t = travel_time([-1, -1, 0, 1, 1], [1, 1, 1, 1, 0])
>>> assert isinstance(t, np.ma.MaskedArray)
>>> np.testing.assert_array_equal(t.data[:-1], [2, 1, 0, 1])
>>> np.testing.assert_array_equal(t.mask, [False, False, False, False, True])
```

```
>>> t2 = travel_time([-1, -1, 0, 1, 1], [1, 1, 1, 1, 1e-300])
>>> np.testing.assert_array_equal(t, t2)
```

Mask test

Test that when the mask cuts off the solution, the cut off points are also masked.

```
>>> ma    = np.ma.MaskedArray([1, 1, 1, 0], [False, True, False, False])
>>> d     = distance(ma)
>>> exact = np.ma.MaskedArray([0, 0, 1, 0], [True, True, False, False])
>>> np.testing.assert_array_equal(d.mask, exact.mask)
>>> np.testing.assert_array_equal(d, exact)
```

Circular level set

```
>>> N     = 50
>>> X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r     = 0.5
>>> dx    = 2.0 / (N - 1)
>>> phi   = (X) ** 2 + (Y) ** 2 - r ** 2
>>> d     = distance(phi, dx, self_test=True)
>>> exact = np.sqrt(X ** 2 + Y ** 2) - r
>>> np.testing.assert_allclose(d, exact, atol=dx)
```

Planar level set

```
>>> N         = 50
>>> X, Y      = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> dx        = 2.0 / (N - 1)
>>> phi       = np.ones_like(X)
>>> phi[0, :] = -1
>>> d         = distance(phi, dx, self_test=True)
>>> exact     = Y + 1 - dx / 2.0
>>> np.testing.assert_allclose(d, exact)
```

Masked input

```
>>> N         = 50
>>> X, Y      = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> dx        = 2.0 / (N - 1)
>>> phi       = np.ones_like(X)
>>> phi[0, 0] = -1
>>> mask      = np.logical_and(abs(X) < 0.25, abs(Y) < 0.25)
>>> mphi      = np.ma.MaskedArray(phi.copy(), mask)
>>> d0        = distance(phi, dx, self_test=True)
```

```
>>> d         = distance(mphi, dx, self_test=True)
>>> d0[mask]  = 0
>>> d[mask]   = 0
>>> shadow    = d0 - d
>>> bsh       = abs(shadow) > 0.001
>>> diff      = (bsh).sum()
```

```
>>> assert diff > 635 and diff < 645
```

Test Eikonal solution

```
>>> N     = 50
>>> X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r     = 0.5
>>> dx    = 2.0 / (N - 1)
>>> phi   = (X) ** 2 + (Y) ** 2 - r ** 2
>>> speed = np.ones_like(phi) * 2
>>> t     = travel_time(phi, speed, dx)
>>> exact = 0.5 * np.abs(np.sqrt(X ** 2 + Y ** 2) - 0.5)
```

```
>>> np.testing.assert_allclose(t, exact, atol=dx)
```

Test 1d

```
>>> N          = 100
>>> X          = np.linspace(-1.0, 1.0, N)
>>> dx         = 2.0 / (N - 1)
>>> phi        = np.zeros_like(X)
>>> phi[X < 0] = -1
>>> phi[X > 0] = 1
>>> d          = distance(phi, dx, self_test=True)
```

```
>>> np.testing.assert_allclose(d, X)
```

Test 3d

```
>>> N            = 15
>>> X            = np.linspace(-1, 1, N)
>>> Y            = np.linspace(-1, 1, N)
>>> Z            = np.linspace(-1, 1, N)
>>> phi          = np.ones((N, N, N))
>>> phi[0, 0, 0] = -1.0
>>> dx           = 2.0 / (N - 1)
>>> d            = distance(phi, dx, self_test=True)
```

```
>>> exact        = np.sqrt((X + 1) ** 2 +
...                        (Y + 1)[:, np.newaxis] ** 2 +
...                        (Z + 1)[:, np.newaxis, np.newaxis] ** 2)
```

```
>>> np.testing.assert_allclose(d, exact, atol=dx)
```

Test default dx

```
>>> N     = 50
>>> X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r     = 0.5
>>> phi   = (X) ** 2 + (Y) ** 2 - r ** 2
>>> speed = np.ones_like(phi) * 2
>>> out = travel_time(phi, speed, self_test=True)
```

Test non-square grid and dx different in different directions

```
>>> N      = 50
>>> NX, NY = N, 5 * N
>>> X, Y   = np.meshgrid(np.linspace(-1, 1, NY), np.linspace(-1, 1, NX))
>>> r      = 0.5
>>> phi    = X ** 2 + Y ** 2 - r ** 2
>>> dx     = [2.0 / (NX - 1), 2.0 / (NY - 1)]
>>> d      = distance(phi, dx, self_test=True)
>>> exact  = np.sqrt(X ** 2 + Y ** 2) - r
```

```
>>> np.testing.assert_allclose(d, exact, atol=1.3*max(dx))
```

No zero level set test

```
>>> distance([1, 1], self_test=False)
Traceback (most recent call last):
  ...
ValueError: the array phi contains no zero contour (no zero level set)
```

Shape mismatch test

```
>>> travel_time([-1, 1], [2])
Traceback (most recent call last):
  ...
ValueError: phi and speed must have the same shape
```

Speed wrong type test

```
>>> travel_time([0, 0, 1, 1], 2)
Traceback (most recent call last):
  ...
ValueError: speed must be a 1D to 12-D array of doubles
```

dx mismatch test

```
>>> travel_time([-1, 1], [2, 2], [2, 2, 2, 2])
Traceback (most recent call last):
  ...
ValueError: dx must be of length len(phi.shape)
```

Test c error handling

```
>>> distance([-1, 1], self_test=44)
Traceback (most recent call last):
  ...
ValueError: self_test must be 0 or 1
```

Check array type test

```
>>> distance(np.array(["a", "b"]))
Traceback (most recent call last):
  ...
ValueError: phi must be a 1 to 12-D array of doubles
```

```
>>> from skfmm import heap
>>> h = heap(10,True)
>>> h.push(0,0.2)
0
>>> h.push(1,0.3)
1
```

```
>>> h.push(2,0.1)
2
>>> h.update(1, 0.01)
>>> h.pop()
(1, 0.01)
>>> h.pop()
(2, 0.1)
>>> h.pop()
(0, 0.2)
>>> h.empty()
True
>>> h.pop()
Traceback (most recent call last):
  ...
RuntimeError: heap pop error: empty heap
```

Test narrow optional argument.

```
>>> phi = np.array([-1,-1,-1,1,1,1])
>>> d = distance(phi, narrow=1.0)
>>> d
masked_array(data = [-- -- -0.5 0.5 -- --],
             mask = [ True  True False False  True  True],
       fill_value = 1e+20)

>>> N     = 50
>>> X, Y  = np.meshgrid(np.linspace(-1, 1, N), np.linspace(-1, 1, N))
>>> r     = 0.5
>>> dx    = 2.0 / (N - 1)
>>> phi   = (X) ** 2 + (Y) ** 2 - r ** 2
>>> exact = np.sqrt(X ** 2 + Y ** 2) - r
```

```
>>> d = distance(phi, dx)
>>> bandwidth=5*dx
>>> d2 = distance(phi, dx, narrow=bandwidth)
>>> np.testing.assert_allclose(d, exact, atol=dx)
>>> # make sure we get a normal array if there are no points outside the narrow band
>>> np.testing.assert_allclose(distance(phi, dx, narrow=1000*dx), exact, atol=dx)
```

```
>>> assert  (d2<bandwidth).all()
>>> assert type(d2) is np.ma.masked_array
```

```
>>> speed = np.ones_like(phi)
>>> speed[X>0] = 1.8
>>> t = travel_time(phi, speed, dx)
>>> t2 = travel_time(phi, speed, dx, narrow=bandwidth)
>>> assert type(t2) is np.ma.masked_array
>>> assert (t2<bandwidth).all()
```

```
>>> speed = np.ones_like(phi)
>>> speed = X + Y
>>> ed, ev = extension_velocities(phi, speed, dx)
>>> ed2, ev2 = extension_velocities(phi, speed, dx, narrow=bandwidth)
>>> assert type(ed2) is np.ma.masked_array
>>> assert type(ev2) is np.ma.masked_array
>>> assert (ed2<bandwidth).all()
>>> assert ed2.shape == ed.shape == ev.shape == ev2.shape
>>> distance(phi, dx, narrow=-1)
```

```
    Traceback (most recent call last):
      ...
    ValueError: parameter "narrow" must be greater than or equal to zero.
    >>> # make sure the narrow options works with an existing mask.
    >>> mask = abs(X)<0.25
    >>> d3 = distance(np.ma.masked_array(phi, mask), dx, narrow=bandwidth)
    >>> assert (d3.mask[mask]==True).all()
    >>> assert d3.mask.sum() > mask.sum()
```

Testing periodic argument

```
    >>> X, Y = np.meshgrid(np.linspace(-2,2,200), np.linspace(-2,2,200))
    >>> phi = -1*np.ones_like(X); phi[X**2+(Y-0.9)**2<0.5] = 1.0
    >>> speed = np.ones_like(X); speed[(X-0.9)**2+Y**2<1.0] = 2.0
    >>> np.allclose(distance(phi),distance(phi,periodic=False)) and np.allclose(distance(phi),distan
    True
    >>> np.allclose(travel_time(phi,speed),travel_time(phi,speed,periodic=False)) and np.allclose(tr
    True
    >>> phi = -1*np.ones_like(X); phi[X**2+Y**2<0.5] = 1.0
    >>> speed = np.ones_like(X); speed[X**2+Y**2<1.0] = 2.0
    >>> np.allclose(distance(phi),distance(phi,periodic=True)) and np.allclose(distance(phi),distanc
    True
    >>> np.allclose(travel_time(phi,speed),travel_time(phi,speed,periodic=True)) and np.allclose(tra
    True
    >>> np.allclose(travel_time(phi,speed),travel_time(phi,speed,periodic=True)) and np.allclose(tra
    True
    >>> np.allclose(travel_time(phi,speed),travel_time(phi,speed,periodic=True)) and np.allclose(tra
    True
    >>> phi = -1*np.ones_like(X); phi[X**2+(Y-0.9)**2<0.5] = 1.0
    >>> speed = np.ones_like(X); speed[(X-0.9)**2+Y**2<1.0] = 2.0
    >>> np.allclose(distance(np.roll(phi,137,axis=0),periodic=True),np.roll(distance(phi,periodic=Tr
    True
    >>> np.allclose(travel_time(np.roll(phi,-77,axis=1),np.roll(speed,-77,axis=1),periodic=True),np.
    True
```

```
    >>> phi=[1,-1,1,1,1,1]
    >>> speed=[4,1,2,2,2,2]
    >>> np.allclose(extension_velocities(phi,speed)[1],(2.5,2.5,1.5,1.5,1.5,1.5))
    True
    >>> np.allclose(extension_velocities(phi,speed,periodic=True)[1],(2.5,2.5,1.5,1.5,1.5,2.5))
    True
```

CHAPTER 2

Limitations:

`scikit-fmm` only works for regular Cartesian grids, but grid cells may have a different (uniform) length in each dimension.

CHAPTER 3

Function Reference

skfmm.**distance**(*phi*, *dx=1.0*, *self_test=False*, *order=2*, *narrow=0.0*, *periodic=False*)

Return the signed distance from the zero contour of the array phi.

Parameters **phi** : array-like

the zero contour of this array is the boundary location for the distance calculation. Phi can of 1,2,3 or higher dimension and can be a masked array.

dx : float or an array-like of len phi.ndim, optional

the cell length in each dimension.

self_test : bool, optional

if True consistency checks are made on the binary min heap during the calculation. This is used in testing and results in a slower calculation.

order : int, optional

order of computational stencil to use in updating points during the fast marching method. Must be 1 or 2, the default is 2.

narrow : float, optional

narrow band half-width. If this optional argument is specified the marching algorithm is limited to within a given narrow band. If far-field points remain when this condition is met a masked array is returned. The default value is 0.0 which means no narrow band limit.

periodic : bool or an array-like of len phi.ndim, optional

specifies whether and in which directions periodic boundary conditions are used. True sets periodic boundary conditions in all directions. An array-like (interpreted as True or False values) specifies the absence or presence of periodic boundaries in individual directions. The default value is False, i.e., no periodic boundaries in any direction.

Returns **d** : an array the same shape as phi

contains the signed distance from the zero contour (zero level set) of phi to each point in the array. The sign is specified by the sign of phi at the given point.

skfmm.**travel_time**(*phi*, *speed*, *dx=1.0*, *self_test=False*, *order=2*, *narrow=0.0*, *periodic=False*)

Return the travel from the zero contour of the array phi given the scalar velocity field speed.

Parameters **phi** : array-like

the zero contour of this array is the boundary location for the travel time calculation. Phi can of 1,2,3 or higher dimension and can be a masked array.

speed : array-like, the same shape as phi

contains the speed of interface propagation at each point in the domain.

dx : float or an array-like of len phi.ndim, optional

the cell length in each dimension.

self_test : bool, optional

if True consistency checks are made on the binary min heap during the calculation. This is used in testing and results in a slower calculation.

order : int, optional

order of computational stencil to use in updating points during the fast marching method. Must be 1 or 2, the default is 2.

narrow : float, optional

narrow band half-width. If this optional argument is specified the marching algorithm is limited to travel times within a given value. If far-field points remain when this condition is met a masked array is returned. The default value is 0.0 which means no narrow band limit.

periodic : bool or an array-like of len phi.ndim, optional

specifies whether and in which directions periodic boundary conditions are used. True sets periodic boundary conditions in all directions. An array-like (interpreted as True or False values) specifies the absence or presence of periodic boundaries in individual directions. The default value is False, i.e., no periodic boundaries in any direction.

Returns **t** : an array the same shape as phi

contains the travel time from the zero contour (zero level set) of phi to each point in the array given the scalar velocity field speed. If the input array speed has values less than or equal to zero the return value will be a masked array.

skfmm.**extension_velocities**(*phi*, *speed*, *dx=1.0*, *self_test=False*, *order=2*, *ext_mask=None*, *narrow=0.0*, *periodic=False*)

Extend the velocities defined at the zero contour of phi, in the normal direction, to the rest of the domain. Extend the velocities such that grad f_ext dot grad d = 0 where where f_ext is the extension velocity and d is the signed distance function.

Parameters **phi** : array-like

the zero contour of this array is the boundary location for the travel time calculation. Phi can of 1,2,3 or higher dimension and can be a masked array.

speed : array-like, the same shape as phi

contains the speed of interface propagation at each point in the domain.

dx : float or an array-like of len phi.ndim, optional

the cell length in each dimension.

self_test : bool, optional

if True consistency checks are made on the binary min heap during the calculation. This is used in testing and results in a slower calculation.

order : int, optional

order of computational stencil to use in updating points during the fast marching method. Must be 1 or 2, the default is 2.

ext_mask : array-like, the same shape as phi, optional

enables initial front values to be eliminated when calculating the value at the interface before the values are extended away from the interface.

narrow : float, optional

narrow band half-width. If this optional argument is specified the marching algorithm is limited to within a given narrow band. If far-field points remain when this condition is met a masked arrays are returned. The default value is 0.0 which means no narrow band limit.

periodic : bool or an array-like of len phi.ndim, optional

specifies whether and in which directions periodic boundary conditions are used. True sets periodic boundary conditions in all directions. An array-like (interpreted as True or False values) specifies the absence or presence of periodic boundaries in individual directions. The default value is False, i.e., no periodic boundaries in any direction.

Returns (d, f_ext) : tuple

a tuple containing the signed distance function d and the extension velocities f_ext.

class skfmm.**heap**(*max_size*, *self_test=False*)

Note: Using this class is not required to use *distance()*, *travel_time()* or *extension_velocities()*. It is provided for experimenting with fast marching algorithms.

This class implements a binary min heap (or heap queue) to support the fast marching algorithm. A min heap is a list which has the property that the smallest element is always the first element. http://en.wikipedia.org/wiki/Binary_heap

This class differs from the heap queue (heapq) in the Python standard library because it supports changing the value of elements in the heap and maintains forward and backward ids.

The fast marching method uses this data structure to track elements in the solution narrow band. The fast marching method needs to know which element in the narrow band is nearest the zero level-set at each iteration. When a new point enters the solution narrow band the element is added to the heap.

New elements (an address and an initial value) are added to the heap with the *push()* method. The *push()* method returns an integer (a heap_id) which is used to update the value of the element.

As the solution evolves, the distance of points already in the heap are updated via the *update()* method. The *update()* method takes the heap_id returned by the *push()* along with a new distance value.

The smallest element is taken off the heap with the *pop()* method. The address and value of the top element is returned.

The constructor for heap needs to know the number of elements that will enter the heap.

```
>>> from skfmm import heap
>>> h = heap(10)
>>> h.push(10,0.2)
0
>>> h.push(11,0.3)
1
>>> h.push(12,0.1)
2
>>> h.peek()
0.1
```

```
>>> h.update(1, 0.01)
>>> h.peek()
0.01
>>> h.pop()
(11, 0.01)
>>> h.pop()
(12, 0.1)
>>> h.pop()
(10, 0.2)
>>> assert h.empty()
```

Methods

empty()

Returns **empty** : Boolean

True if the heap is empty.

peek()

Returns the top (smallest) value on the heap without removing it.

Returns **value** : float

The top (smallest) value on the heap.

pop()

Remove and return the address and value of the top element on the heap.

Returns **(addr, value)** : tuple

A tuple of the address given when the element was pushed onto the heap and the current value of the element.

push(*addr*, *value*)

Add a value to the heap, give an address and a value.

Parameters **addr** : int

An id number which is returned when the element is popped off the heap.

value : float

An initial numerical value for this element.

Returns **heap_id** : int

An id number into the heap used to update the value of this element. The value of a point in the heap can be updated by calling *update()* with this id and a new value.

update(*heap_id*, *value*)

Update the value of a point already in the heap, the heap ordering is updated.

Parameters **heap_id** : int

The heap_id value returned by *push()* when this element was added to the heap.

value : float

The new value for this element.

CHAPTER 4

Testing

To run all the tests use

```
$ python -c "import skfmm; skfmm.test()"
```

See the full Doctests.

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