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# **SafeMDP Documentation**

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## API Documentation

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The *safempd* package implements tools for safe exploration in finite MDPs.

### 1.1 Main classes

These classes provide the main functionality for the safe exploration

<i>SafeMDP</i> (graph, gp, S_hat0, h, L[, beta])	Base class for safe exploration in MDPs.
<i>link_graph_and_safe_set</i> (graph, safe_set)	Link the safe set to the graph model.
<i>reachable_set</i> (graph, initial_nodes[, out])	Compute the safe, reachable set of a graph
<i>returnable_set</i> (graph, reverse_graph, ...[, out])	Compute the safe, returnable set of a graph

#### 1.1.1 SafeMDP

**class** `safempd.SafeMDP` (*graph, gp, S\_hat0, h, L, beta=2*)

Base class for safe exploration in MDPs.

This class only provides basic options to compute the safely reachable and returnable sets. The actual update of the safety feature must be done in a class that inherits from *SafeMDP*. See *safempd.GridWorld* for an example.

**Parameters** **graph:** `networkx.DiGraph`

The graph that models the MDP. Each edge has an attribute *safe* in its metadata, which determines the safety of the transition.

**gp:** `GPy.core.GPRegression`

A Gaussian process model that can be used to determine the safety of transitions. Exact structure depends heavily on the usecase.

**S\_hat0:** **boolean array**

An array that has True on the *i*th position if the *i*th node in the graph is part of the safe set.

**h:** **float**

The safety threshold.

**L:** **float**

The lipschitz constant

**beta: float, optional**

The confidence interval used by the GP model.

### Methods

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<code>add_gp_observations(x_new, y_new)</code>	Add observations to the gp mode.
<code>compute_S_hat()</code>	Compute the safely reachable set given the current safe_set.

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**add\_gp\_observations** (*x\_new*, *y\_new*)

Add observations to the gp mode.

**compute\_S\_hat** ()

Compute the safely reachable set given the current safe\_set.

### 1.1.2 link\_graph\_and\_safe\_set

`safemdp.link_graph_and_safe_set` (*graph*, *safe\_set*)

Link the safe set to the graph model.

**Parameters** *graph*: `nx.DiGraph()`

**safe\_set**: `np.array`

Safe set. For each node the edge (i, j) under action (a) is linked to safe\_set[i, a]

### 1.1.3 reachable\_set

`safemdp.reachable_set` (*graph*, *initial\_nodes*, *out=None*)

Compute the safe, reachable set of a graph

**Parameters** *graph*: `nx.DiGraph`

Directed graph. Each edge must have associated action metadata, which specifies the action that this edge corresponds to. Each edge has an attribute ['safe'], which is a boolean that indicates safety

**initial\_nodes**: `list`

List of the initial, safe nodes that are used as a starting point to compute the reachable set.

**out**: `np.array`

The array to write the results to. Is assumed to be False everywhere except at the initial nodes

**Returns** *reachable\_set*: `np.array`

Boolean array that indicates whether a node belongs to the reachable set.

### 1.1.4 returnable\_set

`safemdp.returnable_set` (*graph*, *reverse\_graph*, *initial\_nodes*, *out=None*)

Compute the safe, returnable set of a graph

**Parameters graph: nx.DiGraph**

Directed graph. Each edge must have associated action metadata, which specifies the action that this edge corresponds to. Each edge has an attribute ['safe'], which is a boolean that indicates safety

**reverse\_graph: nx.DiGraph**

The reversed directed graph, *graph.reverse()*

**initial\_nodes: list**

List of the initial, safe nodes that are used as a starting point to compute the returnable set.

**out: np.array**

The array to write the results to. Is assumed to be False everywhere except at the initial nodes

**Returns** returnable\_set: np.array

Boolean array that indicates whether a node belongs to the returnable set.

## 1.2 Grid world

Some additional functionality specific to gridworlds.

<i>GridWorld</i> (gp, world_shape, step_size, beta, ...)	Grid world with Safe exploration
<i>states_to_nodes</i> (states, world_shape, step_size)	Convert physical states to node numbers.
<i>nodes_to_states</i> (nodes, world_shape, step_size)	Convert node numbers to physical states.
<i>draw_gp_sample</i> (kernel, world_shape, step_size)	Draws a sample from a Gaussian process distribution over a user
<i>grid_world_graph</i> (world_size)	Create a graph that represents a grid world.
<i>grid</i> (world_shape, step_size)	Creates grids of coordinates and indices of state space
<i>compute_true_safe_set</i> (world_shape, altitude, h)	Computes the safe set given a perfect knowledge of the map
<i>compute_true_S_hat</i> (graph, safe_set, ..., [...])	Compute the true safe set with reachability and returnability.
<i>compute_S_hat0</i> (s, world_shape, n_actions, ...)	Compute a valid initial safe seed.
<i>shortest_path</i> (source, next_sample, G)	Computes shortest safe path from a source to the next state-action pair
<i>path_to_boolean_matrix</i> (path, graph, S)	Computes a S-like matrix for approaches where performances is based on
<i>safe_subpath</i> (path, altitudes, h)	Computes the maximum subpath of path along which the safety constraint

### 1.2.1 GridWorld

**class** safemdp.**GridWorld**(gp, world\_shape, step\_size, beta, altitudes, h, S0, S\_hat0, L, update\_dist=0)

Grid world with Safe exploration

**Parameters gp: GPy.core.GP**

Gaussian process that expresses our current belief over the safety feature

**world\_shape: shape**

Tuple that contains the shape of the grid world n x m

**step\_size: tuple of floats**

Tuple that contains the step sizes along each direction to create a linearly spaced grid

**beta: float**

Scaling factor to determine the amplitude of the confidence intervals

**altitudes: np.array**

It contains the flattened  $n \times m$  matrix where the altitudes of all the points in the map are stored

**h: float**

Safety threshold

**S0: np.array**

$n_{\text{states}} \times (n_{\text{actions}} + 1)$  array of booleans that indicates which states (first column) and which state-action pairs belong to the initial safe seed. Notice that, by convention we initialize all the states to be safe

**S\_hat0: np.array or nan**

$n_{\text{states}} \times (n_{\text{actions}} + 1)$  array of booleans that indicates which states (first column) and which state-action pairs belong to the initial safe seed and satisfy recovery and reachability properties. If it is nan, such a boolean matrix is computed during initialization

**noise: float**

Standard deviation of the measurement noise

**L: float**

Lipschitz constant to compute expanders

**update\_dist: int**

Distance in unweighted graph used for confidence interval update. A sample will only influence other nodes within this distance.

**Methods**

<code>add_gp_observations(x_new, y_new)</code>	Add observations to the gp mode.
<code>add_observation(node, action)</code>	Add an observation of the given state-action pair.
<code>compute_S_hat()</code>	Compute the safely reachable set given the current safe_set.
<code>compute_expanders()</code>	Compute the expanders based on the current estimate of S_hat.
<code>plot_S(safe_set[, action])</code>	Plot the set of safe states
<code>target_sample()</code>	Compute the next target (s, a) to sample (highest uncertainty within
<code>update_confidence_interval([jacobian])</code>	Updates the lower and the upper bound of the confidence intervals
<code>update_sets()</code>	Update the sets S, S_hat and G taking with the available observation

**add\_gp\_observations** (*x\_new*, *y\_new*)

Add observations to the gp mode.

**add\_observation** (*node*, *action*)

Add an observation of the given state-action pair.

Observing the pair (s, a) means adding an observation of the altitude at s and an observation of the altitude at f(s, a)

**Parameters node: int**

Node index



**action: int**

Action index

**compute\_S\_hat ()**

Compute the safely reachable set given the current safe\_set.

**compute\_expanders ()**

Compute the expanders based on the current estimate of S\_hat.

**plot\_S (safe\_set, action=0)**

Plot the set of safe states

**Parameters safe\_set: np.array(dtype=bool)**

n\_states x (n\_actions + 1) array of boolean values that indicates the safe set

**action: int**

The action for which we want to plot the safe set.

**target\_sample ()**

Compute the next target (s, a) to sample (highest uncertainty within G or S\_hat)

**Returns** node: int

The next node to sample

action: int

The next action to sample

**update\_confidence\_interval (jacobian=False)**

Updates the lower and the upper bound of the confidence intervals using then posterior distribution over the gradients of the altitudes

**Returns** l: np.array

lower bound of the safety feature (mean - beta\*std)

u: np.array

upper bound of the safety feature (mean + beta\*std)

**update\_sets ()**

Update the sets S, S\_hat and G taking with the available observation

## 1.2.2 states\_to\_nodes

`safemdp.states_to_nodes (states, world_shape, step_size)`

Convert physical states to node numbers.

**Parameters states: np.array**

States with physical coordinates

**world\_shape: tuple**

The size of the grid\_world

**step\_size: tuple**

The step size of the grid world

**Returns** nodes: np.array

The node indices corresponding to the states

### 1.2.3 nodes\_to\_states

`safemdp.nodes_to_states(nodes, world_shape, step_size)`

Convert node numbers to physical states.

**Parameters** **nodes: np.array**

Node indices of the grid world

**world\_shape: tuple**

The size of the grid\_world

**step\_size: np.array**

The step size of the grid world

**Returns** **states: np.array**

The states in physical coordinates

### 1.2.4 draw\_gp\_sample

`safemdp.draw_gp_sample(kernel, world_shape, step_size)`

Draws a sample from a Gaussian process distribution over a user specified grid

**Parameters** **kernel: GP kernel**

Defines the GP we draw a sample from

**world\_shape: tuple**

Shape of the grid we use for sampling

**step\_size: tuple**

Step size along any axis to find linearly spaced points

### 1.2.5 grid\_world\_graph

`safemdp.grid_world_graph(world_size)`

Create a graph that represents a grid world.

In the grid world there are four actions, (1, 2, 3, 4), which correspond to going (up, right, down, left) in the x-y plane. The states are ordered so that `np.arange(np.prod(world_size)).reshape(world_size)` corresponds to a matrix where increasing the row index corresponds to the x direction in the graph, and increasing y index corresponds to the y direction.

**Parameters** **world\_size: tuple**

The size of the grid world (rows, columns)

**Returns** **graph: nx.DiGraph()**

The directed graph representing the grid world.

### 1.2.6 grid

`safemdp.grid(world_shape, step_size)`

Creates grids of coordinates and indices of state space

**Parameters world\_shape: tuple**

Size of the grid world (rows, columns)

**step\_size: tuple**

Physical step size in the grid world

**Returns states\_ind: np.array**

(n\*m) x 2 array containing the indices of the states

**states\_coord: np.array**

(n\*m) x 2 array containing the coordinates of the states

### 1.2.7 compute\_true\_safe\_set

`safemdp.compute_true_safe_set` (*world\_shape, altitude, h*)

Computes the safe set given a perfect knowledge of the map

**Parameters world\_shape: tuple****altitude: np.array**

1-d vector with altitudes for each node

**h: float**

Safety threshold for height differences

**Returns true\_safe: np.array**

Boolean array n\_states x (n\_actions + 1).

### 1.2.8 compute\_true\_S\_hat

`safemdp.compute_true_S_hat` (*graph, safe\_set, initial\_nodes, reverse\_graph=None*)

Compute the true safe set with reachability and returnability.

**Parameters graph: nx.DiGraph****safe\_set: np.array****initial\_nodes: list of int****reverse\_graph: nx.DiGraph**

`graph.reverse()`

**Returns true\_safe: np.array**

Boolean array n\_states x (n\_actions + 1).

### 1.2.9 compute\_S\_hat0

`safemdp.compute_S_hat0` (*s, world\_shape, n\_actions, altitudes, step\_size, h*)

Compute a valid initial safe seed.

**s: int or nan** Vector index of the state where we start computing the safe seed from. If it is equal to nan, a state is chosen at random

**world\_shape: tuple** Size of the grid world (rows, columns)

**n\_actions: int** Number of actions available to the agent

**altitudes: np.array** It contains the flattened  $n \times m$  matrix where the altitudes of all the points in the map are stored

**step\_size: tuple** step sizes along each direction to create a linearly spaced grid

**h: float** Safety threshold

**S\_hat: np.array** Boolean array  $n\_states \times (n\_actions + 1)$ .

### 1.2.10 shortest\_path

`safemdp.shortest_path` (*source*, *next\_sample*, *G*)

Computes shortest safe path from a source to the next state-action pair the agent needs to sample

**Parameters** **source: int**

Starting node for the path

**next\_sample: (int, int)**

Next state-action pair the agent needs to sample. First entry is the number that indicates the state. Second entry indicates the action

**G: networkx DiGraph**

Graph that indicates the dynamics. It is linked to S matrix

**Returns** path: list

shortest safe path

### 1.2.11 path\_to\_boolean\_matrix

`safemdp.path_to_boolean_matrix` (*path*, *graph*, *S*)

Computes a S-like matrix for approaches where performances is based on the trajectory of the agent (e.g. unsafe or random exploration) Parameters ——— path: np.array

Contains the nodes that are visited along the path

**graph: networkx.DiGraph** Graph that indicates the dynamics

**S: np.array** Array describing the safe set (needed for initialization)

**Returns** bool\_mat: np.array

S-like array that is true for all the states and state-action pairs along the path

### 1.2.12 safe\_subpath

`safemdp.safe_subpath` (*path*, *altitudes*, *h*)

Computes the maximum subpath of path along which the safety constraint is not violated Parameters ——— path: np.array

Contains the nodes that are visited along the path

**altitudes: np.array** 1-d vector with altitudes for each node

**h: float** Safety threshold

**Returns** subpath: np.array

Maximum subpath of path that fulfills the safety constraint

## 1.3 Utilities

The following are utilities to make testing and working with the library more pleasant.

<code>DifferenceKernel(kernel)</code>	A fake kernel that can be used to predict differences two function values.
<code>max_out_degree(graph)</code>	Compute the maximum out_degree of a graph

### 1.3.1 DifferenceKernel

**class** `safemdp.DifferenceKernel` (*kernel*)

A fake kernel that can be used to predict differences two function values.

Given a gp based on measurements, we aim to predict the difference between the function values at two different test points, X1 and X2; that is, we want to obtain mean and variance of  $f(X1) - f(X2)$ . Using this fake kernel, this can be achieved with `mean, var = gp.predict(np.hstack((X1, X2)), kern=DiffKernel(gp.kern))`

**Parameters** **kernel: GPy.kern.\***

The kernel used by the GP

#### Methods

<code>K(x1[, x2])</code>	Equivalent of kern.K
<code>Kdiag(x)</code>	Equivalent of kern.Kdiag for the difference prediction.

**K** (*x1, x2=None*)

Equivalent of kern.K

If only x1 is passed then it is assumed to contain the data for both whose differences we are computing. Otherwise, x2 will contain these extended states (see `PosteriorExact._raw_predict` in `GPpy/inference/latent_function_inference0/posterior.py`)

**Parameters** **x1: np.array**

**x2: np.array**

**Kdiag** (*x*)

Equivalent of kern.Kdiag for the difference prediction.

**Parameters** **x: np.array**

### 1.3.2 max\_out\_degree

`safemdp.max_out_degree` (*graph*)

Compute the maximum out\_degree of a graph

**Parameters** **graph: nx.DiGraph**

**Returns** max\_out\_degree: int

The maximum out\_degree of the graph

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