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# **fpyll Documentation**

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A Python (2 and 3) wrapper for fplll.

```
>>> from fpylll import *
>>> A = IntegerMatrix(50, 50)
>>> A.randomize("ntrulike", bits=50, q=127)
>>> A[0].norm()
3564748886669202.5
>>> M = GSO.Mat(A)
>>> M.update_gso()
>>> M.get_mu(1, 0)
0.815748944429783
>>> L = LLL.Reduction(M)
>>> L()
>>> M.get_mu(1, 0)
0.41812865497076024
>>> A[0].norm()
24.06241883103193
```

The basic BKZ algorithm can be implemented in about 60 pretty readable lines of Python code (cf. [simple\\_bkz.py](#)).

## Requirements

fpylll relies on the following C/C++ libraries:

- [GMP](#) or [MPPIR](#) for arbitrary precision integer arithmetic.
- [MPFR](#) for arbitrary precision floating point arithmetic.
- [QD](#) for double double and quad double arithmetic (optional).
- [fplll](#) for pretty much everything.

fpyll also relies on

- [Cython](#) for linking Python and C/C++.
- [cysignals](#) for signal handling such as interrupting C++ code.
- [py.test](#) for testing Python.
- [flake8](#) for linting.

We also suggest

- [IPython](#) for interacting with Python
- [Numpy](#) for numerical computations (e.g. with Gram-Schmidt values)

## Online

fpyll ships with Sage 7.4. Thus, it is available via [SageMathCell](#) and [SageMathCloud](#) (select a Jupyter notebook with a Sage 7.4 kernel, the default Sage worksheet still runs Sage 7.3 at the time of writing). You can also fire up a [dply.co virtual server](#) with the latest fpyll/fplll preinstalled (it takes perhaps 15 minutes until everything is compiled).

## Getting Started

**Note:** fpyll is also available via [PyPI](#) and [Conda-Forge](#) for [Conda](#). In what follows, we explain manual installation.

We recommend [virtualenv](#) for isolating Python build environments and [virtualenvwrapper](#) to manage virtual environments.

1. Create a new virtualenv and activate it:

```
$ virtualenv env
$ source ./env/bin/activate
```

We indicate active virtualenvs by the prefix (fpylll).

2. Install the required libraries - [GMP](#) or [MPIR](#) and [MPFR](#) - if not available already. You may also want to install [QD](#).
3. Install fplll:

```
$ (fpylll) ./install-dependencies.sh $VIRTUAL_ENV
```

4. Then, execute:

```
$ (fpylll) pip install Cython
$ (fpylll) pip install -r requirements.txt
```

to install the required Python packages (see above).

5. If you are so inclined, run:

```
$ (fpylll) pip install -r suggestions.txt
```

to install suggested Python packages as well (optional).

6. Build the Python extension:

```
$ (fpylll) export PKG_CONFIG_PATH="$VIRTUAL_ENV/lib/pkgconfig:$PKG_CONFIG_PATH"
$ (fpylll) python setup.py build_ext
$ (fpylll) python setup.py install
```

7. To run **fpylll**, you will need to:

```
$ (fpylll) export LD_LIBRARY_PATH="$VIRTUAL_ENV/lib"
```

so that Python can find **fpdll** and friends.

8. Start Python:

```
$ (fpylll) ipython
```

To reactivate the virtual environment later, simply run:

```
$ source ./env/bin/activate
```

Note that you can also patch `activate` to set `LD_LIBRARY_PATH`. For this, add:

```
### LD_LIBRARY_HACK
_OLD_LD_LIBRARY_PATH="$LD_LIBRARY_PATH"
LD_LIBRARY_PATH="$VIRTUAL_ENV/lib:$LD_LIBRARY_PATH"
export LD_LIBRARY_PATH
### END_LD_LIBRARY_HACK

### PKG_CONFIG_HACK
_OLD_PKG_CONFIG_PATH="$PKG_CONFIG_PATH"
PKG_CONFIG_PATH="$VIRTUAL_ENV/lib/pkgconfig:$PKG_CONFIG_PATH"
export PKG_CONFIG_PATH
### END_PKG_CONFIG_HACK
```

towards the end and:

```
### LD_LIBRARY_HACK
if ! [ -z ${_OLD_LD_LIBRARY_PATH+x} ] ; then
    LD_LIBRARY_PATH=$_OLD_LD_LIBRARY_PATH
    export LD_LIBRARY_PATH
    unset _OLD_LD_LIBRARY_PATH
fi
### END_LD_LIBRARY_HACK

### PKG_CONFIG_HACK
if ! [ -z ${_OLD_PKG_CONFIG_PATH+x} ] ; then
    PKG_CONFIG_PATH=$_OLD_PKG_CONFIG_PATH
    export PKG_CONFIG_PATH
    unset _OLD_PKG_CONFIG_PATH
fi
### END_PKG_CONFIG_HACK
```

in the `deactivate` function in the `activate` script.

## Multicore Support

**fpylll** supports parallelisation on multiple cores. For all C++ support to drop the GIL is enabled, allowing the use of threads to parallelise. **Fpdll** is thread safe as long as each thread works on a separate object such as `IntegerMatrix`

or `MatGSO`. Also, **fpylll** does not actually drop the GIL in all calls to C++ functions yet. In many scenarios using [multiprocessing](#), which sidesteps the GIL and thread safety issues by using processes instead of threads, will be the better choice.

The example below calls `LLL.reduction` on 128 matrices of dimension 30 on four worker processes.

```
from fpylll import IntegerMatrix, LLL
from multiprocessing import Pool

d, workers, tasks = 30, 4, 128

def run_it(p, f, A, prefix=""):
    """Print status during parallel execution."""
    import sys
    r = []
    for i, retval in enumerate(p.imap_unordered(f, A, 1)):
        r.append(retval)
        sys.stderr.write('\r{0} done: {1:.2%}'.format(prefix, float(i)/len(A)))
        sys.stderr.flush()
    sys.stderr.write('\r{0} done {1:.2%}\n'.format(prefix, float(i+1)/len(A)))
    return r

A = [IntegerMatrix.random(d, "uniform", bits=30) for _ in range(tasks)]
A = run_it(Pool(workers), LLL.reduction, A)
```

To test threading simply replace the line `from multiprocessing import Pool` with `from multiprocessing.pool import ThreadPool as Pool`. For calling `BKZ.reduction` this way, which expects a second parameter with options, using `functools.partial` is a good choice.

## Contributing

**fpylll** welcomes contributions, cf. the list of [open issues](#). To contribute, clone this repository, commit your code on a separate branch and send a pull request. Please write tests for your code. You can run them by calling:

```
$ (fpylll) py.test
```

from the top-level directory which runs all tests in `tests/test_*.py`. We run [flake8](#) on every commit automatically. In particular, we run:

```
$ (fpylll) flake8 --max-line-length=120 --max-complexity=16 --ignore=E22,E241 src
```

Note that **fpylll** supports Python 2 and 3. In particular, tests are run using Python 2.7 and 3.5. See [.travis.yml](#) for details on automated testing.

## Attribution & License

**fpylll** is maintained by Martin Albrecht.

The following people have contributed to **fpylll**

- Martin Albrecht
- Guillaume Bonnoron
- Jeroen Demeyer

- Leo Ducas
- Omer Katz

We copied a decent bit of code over from Sage, mostly from it's fpLLL interface.

**fpyll** is licensed under the GPLv2+.



## fpIII Modules

The modules in this category in some way represent classes or functions from fpIII. They are typically implemented in Cython.

### Integer Matrices

Integer matrices.

**class** `fpIII.fpIII.integer_matrix.IntegerMatrix` (*arg0*, *arg1=None*)  
Dense matrices over the Integers.

`__copy__` (*self*)  
Copy this matrix.

`__getitem__`  
Select a row or entry.

**Parameters** **key** – an integer for the row, a tuple for row and column or a slice.

**Returns** a reference to a row or an integer depending on format of **key**

```
>>> from fpIII import IntegerMatrix
>>> A = IntegerMatrix(10, 10)
>>> A.gen_identity(10)
>>> A[1,0]
0
```

```
>>> print(A[1])
(0, 1, 0, 0, 0, 0, 0, 0, 0, 0)
```

```
>>> print(A[0:2])
[ 1 0 0 0 0 0 0 0 0 0 ]
[ 0 1 0 0 0 0 0 0 0 0 ]
```

### \_\_init\_\_

Construct a new integer matrix

#### Parameters

- **arg0** – number of rows 0 or matrix
- **arg1** – number of columns 0 or None

The default constructor takes the number of rows and columns:

```
>>> from fpylll import IntegerMatrix
>>> IntegerMatrix(10, 10)
<IntegerMatrix(10, 10) at 0x...>

>>> IntegerMatrix(10, 0)
<IntegerMatrix(10, 0) at 0x...>

>>> IntegerMatrix(-1, 0)
Traceback (most recent call last):
...
ValueError: Number of rows must be >0
```

The default constructor is also a copy constructor:

```
>>> A = IntegerMatrix(2, 2)
>>> A[0,0] = 1
>>> B = IntegerMatrix(A)
>>> B[0,0]
1
>>> A[0,0] = 2
>>> B[0,0]
1
```

### \_\_setitem\_\_

Assign value to index.

#### Parameters

- **key** – a tuple of row and column indices
- **value** – an integer

EXAMPLE:

```
>>> from fpylll import IntegerMatrix
>>> A = IntegerMatrix(10, 10)
>>> A.gen_identity(10)
>>> A[1,0] = 2
>>> A[1,0]
2
```

The notation `A[i][j]` is not supported. This is because `A[i]` returns an object of type `IntegerMatrixRow` object which is immutable by design. This is to avoid the user confusing such an object with a proper vector.:

```
>>> A[1][0] = 2
Traceback (most recent call last):
...
TypeError: 'fpvlll.fplll.integer_matrix.IntegerMatrixRow' object does not
↳support item assignment
```

**apply\_transform** (*self*, *IntegerMatrix U*, *int start\_row=0*)  
Apply transformation matrix *U* to this matrix starting at row *start\_row*.

**Parameters**

- **U** (*IntegerMatrix*) – transformation matrix
- **start\_row** (*int*) – start transformation in this row

**clear** (*self*)

**classmethod from\_file** (*type cls*, *filename*)  
Construct new matrix from file.

**Parameters filename** – name of file to read from

**classmethod from\_iterable** (*type cls*, *nrows*, *ncols*, *it*)  
Construct a new integer matrix from matrix-like object *A*

**Parameters**

- **nrows** – number of rows
- **ncols** – number of columns
- **it** – an iterable of length at least *nrows \* ncols*

```
>>> A = IntegerMatrix.from_iterable(2, 3, [1, 2, 3, 4, 5, 6])
>>> print(A)
[ 1 2 3 ]
[ 4 5 6 ]
```

**classmethod from\_matrix** (*type cls*, *A*, *nrows=None*, *ncols=None*)  
Construct a new integer matrix from matrix-like object *A*

**Parameters**

- **A** – a matrix like object, with element access *A*[*i*,*j*] or *A*[*i*][*j*]
- **nrows** – number of rows (optional)
- **ncols** – number of columns (optional)

```
>>> A = IntegerMatrix.from_matrix([[1, 2, 3], [4, 5, 6]])
>>> print(A)
[ 1 2 3 ]
[ 4 5 6 ]
```

**gen\_identity** (*self*, *int nrows*)  
Generate identity matrix.

**Parameters nrows** – number of rows

**get\_max\_exp** (*self*)

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
>>> A.get_max_exp()
3
```

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,9]])
>>> A.get_max_exp()
4
```

**classmethod `identity`** (*type cls, n rows*)

Construct a new identity matrix of dimension `n rows` × `n rows`

**Parameters** `n rows` – number of rows.

```
>>> A = IntegerMatrix.identity(4)
>>> print(A)
[ 1 0 0 0 ]
[ 0 1 0 0 ]
[ 0 0 1 0 ]
[ 0 0 0 1 ]
```

**`is_empty`** (*self*)

**mod** (*self, q, int start\_row=0, int start\_col=0, int stop\_row=-1, int stop\_col=-1*)

Apply modular reduction modulo `q` to this matrix.

**Parameters**

- `q` – modulus
- `start_row` (*int*) – starting row
- `start_col` (*int*) – starting column
- `stop_row` (*int*) – last row (excluding)
- `stop_col` (*int*) – last column (excluding)

```
>>> A = IntegerMatrix(2, 2)
>>> A[0,0] = 1001
>>> A[1,0] = 13
>>> A[0,1] = 102
>>> print(A)
[ 1001 102 ]
[ 13 0 ]
```

```
>>> A.mod(10, start_row=1, start_col=0)
>>> print(A)
[ 1001 102 ]
[ 3 0 ]
```

```
>>> A.mod(10)
>>> print(A)
[ 1 2 ]
[ 3 0 ]
```

```
>>> A = IntegerMatrix(2, 2)
>>> A[0,0] = 1001
>>> A[1,0] = 13
>>> A[0,1] = 102
```

```
>>> A.mod(10, stop_row=1)
>>> print(A)
[  1  2 ]
[ 13  0 ]
```

**multiply\_left** (*self*, *v*, *start=0*)

Return  $v * A'$  where  $A'$  is  $A$  reduced to  $\text{len}(v)$  rows starting at *start*.

**Parameters**

- **v** – a tuple-like object
- **start** – start in row *start*

**ncols**

Number of Columns

**Returns** number of columns

```
>>> from fpylll import IntegerMatrix
>>> IntegerMatrix(10, 10).ncols
10
```

**nrows**

Number of Rows

**Returns** number of rows

```
>>> from fpylll import IntegerMatrix
>>> IntegerMatrix(10, 10).nrows
10
```

**classmethod random** (*type cls*, *d*, *algorithm*, *\*\*kws*)

Construct new random matrix.

**Seealso** *IntegerMatrix.randomize*

**randomize** (*self*, *algorithm*, *\*\*kws*)

Randomize this matrix using *algorithm*.

**Parameters** **algorithm** – string, see below for choices.

Available algorithms:

- "intrel" - generate a knapsack like matrix of dimension  $d \times (d+1)$  and *bits* bits: the *i*-th vector starts with a random integer of bit-length  $\leq b$  and the rest is the *i*-th canonical unit vector.
- "simdioph" - generate a  $d \times d$  matrix of a form similar to that is involved when trying to find rational approximations to reals with the same small denominator. The first vector starts with a random integer of bit-length  $\leq \text{bits}2$  and continues with  $d-1$  independent integers of bit-lengths  $\leq \text{bits}$ ; the *i*-th vector for  $i > 1$  is the *i*-th canonical unit vector scaled by a factor  $2^b$ .
- "uniform" - generate a  $d \times d$  matrix whose entries are independent integers of bit-lengths  $\leq \text{bits}$ .
- "ntrulike" - generate an NTRU-like matrix. If *bits* is given, then it first samples an integer *q* of bit-length  $\leq \text{bits}$ , whereas if *q*, then it sets *q* to the provided value. Then it samples a uniform *h* in the ring  $\mathbb{Z}_q[x] / (x^n - 1)$ . It finally returns the  $2 \times 2$  block matrix  $\begin{bmatrix} I & \text{Rot}(h) \\ 0 & q \cdot I \end{bmatrix}$ , where each block is  $d \times d$ , the first row of

$\text{Rot}(h)$  is the coefficient vector of  $h$ , and the  $i$ -th row of  $\text{Rot}(h)$  is the shift of the  $(i-1)$ -th (with last entry put back in first position), for all  $i > 1$ . Warning: this does not produce a genuine ntru lattice with  $h$  a genuine public key.

- `ntrulike2` : as the previous option, except that the constructed matrix is  $\begin{bmatrix} [q \cdot I, \\ 0], [\text{Rot}(h), I] \end{bmatrix}$ .
- `"qary"` : generate a  $q$ -ary matrix. If `bits` is given, then it first samples an integer  $q$  of bit-length  $\leq \text{bits}$ ; if  $q$  is provided, then set  $q$  to the provided value. It returns a  $2 \times 2$  block matrix  $\begin{bmatrix} [q \cdot I, 0], [H, I] \end{bmatrix}$ , where  $H$  is  $k \times (d-k)$  and uniformly random modulo  $q$ . These bases correspond to the SIS/LWE  $q$ -ary lattices. Goldstein-Mayer lattices correspond to  $k=1$  and  $q$  prime.
- `"trg"` - generate a  $d \times d$  lower-triangular matrix  $B$  with  $B_{ii} = 2^{(d-i+1)^f}$  for all  $i$ , and  $B_{ij}$  is uniform between  $-B_{jj}/2$  and  $B_{jj}/2$  for all  $j < i$ .

**resize** (*self*, *int rows*, *int cols*)

**Parameters**

- **rows** (*int*) -
- **cols** (*int*) -

**rotate** (*self*, *int first*, *int middle*, *int last*)

Rotates the order of the elements in the range  $[\text{first}, \text{last})$ , in such a way that the element pointed by `middle` becomes the new first element.

$(M[\text{first}], \dots, M[\text{middle}-1], M[\text{middle}], M[\text{last}])$  becomes  $(M[\text{middle}], \dots, M[\text{last}], M[\text{first}], \dots, M[\text{middle}-1])$

**Parameters**

- **first** (*int*) - first index
- **middle** (*int*) - new first index
- **last** (*int*) - last index (inclusive)

```
>>> A = IntegerMatrix.from_matrix([[0,1,2],[3,4,5],[6,7,8]])
>>> A.rotate(0,0,2)
>>> print(A)
[ 0 1 2 ]
[ 3 4 5 ]
[ 6 7 8 ]
```

```
>>> A = IntegerMatrix.from_matrix([[0,1,2],[3,4,5],[6,7,8]])
>>> A.rotate(0,2,2)
>>> print(A)
[ 6 7 8 ]
[ 0 1 2 ]
[ 3 4 5 ]
```

**rotate\_gram\_left** (*self*, *int first*, *int last*, *int n\_valid\_rows*)

Transformation needed to update the lower triangular Gram matrix when `rotateLeft(first, last)` is done on the basis of the lattice.

**Parameters**

- **first** (*int*) -
- **last** (*int*) -

- **n\_valid\_rows** (*int*) –

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
```

**rotate\_gram\_right** (*self, int first, int last, int n\_valid\_rows*)

Transformation needed to update the lower triangular Gram matrix when `rotateRight(first, last)` is done on the basis of the lattice.

**Parameters**

- **first** (*int*) –
- **last** (*int*) –
- **n\_valid\_rows** (*int*) –

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
```

**rotate\_left** (*self, int first, int last*)

Row permutation.

( $M[\text{first}], \dots, M[\text{last}]$ ) becomes ( $M[\text{first}+1], \dots, M[\text{last}], M[\text{first}]$ )

**Parameters**

- **first** (*int*) –
- **last** (*int*) –

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
```

**rotate\_right** (*self, int first, int last*)

Row permutation.

( $M[\text{first}], \dots, M[\text{last}]$ ) becomes ( $M[\text{last}], M[\text{first}], \dots, M[\text{last}-1]$ )

**Parameters**

- **first** (*int*) –
- **last** (*int*) –

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
```

**set\_cols** (*self, int cols*)

**Parameters** **cols** (*int*) –

**set\_iterable** (*self, A*)

Set this matrix from iterable A

**Parameters** **A** – an iterable object such as a list or tuple

**Warning:** entries starting at  $A[\text{nrows} * \text{ncols}]$  are ignored.

**set\_matrix** (*self, A*)

Set this matrix from matrix-like object A

**Parameters** **A** – a matrix like object, with element access  $A[i,j]$  or  $A[i][j]$

**Warning:** entries starting at A[nrows, ncols] are ignored.

**set\_rows** (*self*, *int rows*)

**Parameters** *rows* (*int*) –

**submatrix** (*self*, *a*, *b*, *c=None*, *d=None*)

Construct a new submatrix.

**Parameters**

- **a** – either the index of the first row or an iterable of row indices
- **b** – either the index of the first column or an iterable of column indices
- **c** – the index of first excluded row (or None)
- **d** – the index of first excluded column (or None)

**Returns**

**Return type**

We illustrate the calling conventions of this function using a 10 x 10 matrix:

```
>>> from fpylll import IntegerMatrix, set_random_seed
>>> A = IntegerMatrix(10, 10)
>>> set_random_seed(1337)
>>> A.randomize("ntrulike", bits=22, q=4194319)
>>> print(A)
[ 1 0 0 0 0 3021421 752690 1522220 2972677 119630 ]
[ 0 1 0 0 0 119630 3021421 752690 1522220 2972677 ]
[ 0 0 1 0 0 2972677 119630 3021421 752690 1522220 ]
[ 0 0 0 1 0 1522220 2972677 119630 3021421 752690 ]
[ 0 0 0 0 1 752690 1522220 2972677 119630 3021421 ]
[ 0 0 0 0 0 4194319 0 0 0 0 ]
[ 0 0 0 0 0 0 4194319 0 0 0 ]
[ 0 0 0 0 0 0 0 4194319 0 0 ]
[ 0 0 0 0 0 0 0 0 4194319 0 ]
[ 0 0 0 0 0 0 0 0 0 4194319 ]
```

We can either specify start/stop rows and columns:

```
>>> print(A.submatrix(0,0,2,8))
[ 1 0 0 0 0 3021421 752690 1522220 ]
[ 0 1 0 0 0 119630 3021421 752690 ]
```

Or we can give lists of rows, columns explicitly:

```
>>> print(A.submatrix([0,1,2], range(3,9)))
[ 0 0 3021421 752690 1522220 2972677 ]
[ 0 0 119630 3021421 752690 1522220 ]
[ 0 0 2972677 119630 3021421 752690 ]
```

**swap\_rows** (*self*, *int r1*, *int r2*)

**Parameters**

- **r1** (*int*) –
- **r2** (*int*) –

```

>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
>>> A.swap_rows(0, 1)
>>> print(A)
[ 3 4 ]
[ 0 2 ]

```

**to\_matrix** (*self*, *A*)

Write this matrix to matrix-like object *A*

**Parameters** **A** – a matrix like object, with element access  $A[i,j]$  or  $A[i][j]$

**Returns** *A*

**transpose** (*self*)

Transpose.

```

>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
>>> _ = A.transpose()
>>> print(A)
[ 0 3 ]
[ 2 4 ]

```

**class** `fpylll.fpylll.integer_matrix.IntegerMatrixRow` (*IntegerMatrix M*, *int row*)  
A reference to a row in an integer matrix.

**\_\_getitem\_\_**

Return entry at *column*

**Parameters** **column** (*int*) – integer offset

**\_\_init\_\_**

Create a row reference.

**Parameters**

- **M** (*IntegerMatrix*) – Integer matrix
- **row** (*int*) – row index

Row references are immutable:

```

>>> from fpylll import IntegerMatrix
>>> A = IntegerMatrix(2, 3)
>>> A[0,0] = 1; A[0,1] = 2; A[0,2] = 3
>>> r = A[0]
>>> r[0]
1
>>> r[0] = 1
Traceback (most recent call last):
...
TypeError: 'fpylll.fpylll.integer_matrix.IntegerMatrixRow' object does not_
↳support item assignment

```

**addmul** (*self*, *IntegerMatrixRow v*, *x=1*, *int expo=0*)

In-place add row vector  $2^{\text{expo}} \times v$

**Parameters**

- **v** (*IntegerMatrixRow*) – row vector
- **x** – multiplier
- **expo** (*int*) – scaling exponent.

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
>>> A[0].addmul(A[1])
>>> print(A[0])
(3, 6)
```

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
>>> A[0].addmul(A[1],x=0)
>>> print(A[0])
(0, 2)
```

```
>>> A = IntegerMatrix.from_matrix([[0,2],[3,4]])
>>> A[0].addmul(A[1],x=1,expo=2)
>>> print(A[0])
(12, 18)
```

**is\_zero** (*self*, *int frm=0*)

Return True if this vector consists of only zeros starting at index *frm*

```
>>> A = IntegerMatrix.from_matrix([[1,0,0]])
>>> A[0].is_zero()
False
>>> A[0].is_zero(1)
True
```

**norm**

Return 2 norm of this vector.

```
>>> A = IntegerMatrix.from_iterable(1, 3, [1,2,3])
>>> A[0].norm()
3.74165...
>>> 1*1 + 2*2 + 3*3
14
>>> from math import sqrt
>>> sqrt(14)
3.74165...
```

**size\_nz** (*self*)

Index at which an all zero vector starts.

```
>>> A = IntegerMatrix.from_matrix([[0,2,3],[0,2,0],[0,0,0]])
>>> A[0].size_nz()
3
>>> A[1].size_nz()
2
>>> A[2].size_nz()
0
```

fpylll.fpylll.integer\_matrix.**unpickle\_IntegerMatrix** (*nrows*, *ncols*, *l*)

Deserialize an integer matrix.

**Parameters**

- **nrows** – number of rows
- **ncols** – number of columns
- **l** – list of entries

## Gram Schmidt Orthogonalization

Elementary basis operations, Gram matrix and Gram-Schmidt orthogonalization.

A `MatGSO` object stores the following information:

- The integral basis  $B$ ,
- the Gram-Schmidt coefficients  $i,j = 'b_i, b_j^* / ||b_j^*||^2$  for  $i > j$ , and
- the coefficients  $r_{i,j} = b_i, b_j^*$  for  $i > j$

It holds that:  $B = RQ = (D)(D^{-1}B^*)$  where  $Q$  is orthonormal and  $R$  is lower triangular.

```
class fpylll.fpylll.gso.GSO
```

```
    DEFAULT = 0
```

```
    INT_GRAM = 1
```

```
    Mat
```

```
        alias of MatGSO
```

```
    OP_FORCE_LONG = 4
```

```
    ROW_EXPO = 2
```

```
class fpylll.fpylll.gso.MatGSO (IntegerMatrix B, U=None, UinvT=None, int flags=GSO_DEFAULT,
                                float_type='double')
```

`MatGSO` provides an interface for performing elementary operations on a basis and computing its Gram matrix and its Gram-Schmidt orthogonalization. The Gram-Schmidt coefficients are computed on demand. The object keeps track of which coefficients are valid after each row operation.

```
    B
```

```
    U
```

```
    UinvT
```

```
    __init__
```

### Parameters

- **B** (`IntegerMatrix`) – The matrix on which row operations are performed. It must not be empty.
- **U** (`IntegerMatrix`) – If **U** is not empty, operations on **B** are also done on **u** (in this case both must have the same number of rows). If **u** is initially the identity matrix, multiplying transform by the initial basis gives the current basis.
- **UinvT** (`IntegerMatrix`) – Inverse transform (should be empty, which disables the computation, or initialized with identity matrix). It works only if **U** is not empty.
- **flags** (`int`) – Flags
  - `GSO.INT_GRAM` - If true, coefficients of the Gram matrix are computed with exact integer arithmetic. Otherwise, they are computed in floating-point. Note that when exact arithmetic is used, all coefficients of the first `n_known_rows` are continuously updated, whereas in floating-point, they are computed only on-demand. This option cannot be enabled when `GSO.ROW_EXPO` is set.
  - `GSO.ROW_EXPO` - If true, each row of **B** is normalized by a power of 2 before doing conversion to floating-point, which hopefully avoids some overflows. This option cannot be enabled if `GSO.INT_GRAM` is set and works only with

`float_type="double"` and `float_type="long double"`. It is useless and **must not** be used for `float_type="dpe"`, `float_type="dd"`, `float_type="qd"` or `float_type=mpfr_t`.

- `GSO.OP_FORCE_LONG` - Affects the behaviour of `row_addmul`. See its documentation.
- **float\_type** - A floating point type, i.e. an element of `fpylll.fpylll.float_types`.

---

**Note:** If `float_type="mpfr"` set precision with `set_precision()` before constructing this object and do not change the precision during the lifetime of this object.

---

**babai** (*self*, *v*, *int start=0*, *int dimension=-1*, *gso=False*)

Return lattice vector close to *v* using Babai's nearest plane algorithm.

#### Parameters

- **v** - a tuple-like object
- **start** - only consider subbasis starting at `start``
- **dimension** - only consider `dimension` vectors or all if `-1``
- **gso** - if `True` vector is represented wrt to the Gram-Schmidt basis, otherwise canonical basis is assumed.

**Returns** a tuple of dimension *M.B.nrows*

**create\_row** (*self*)

Adds a zero row to *B* (and to *U* if `enable_transform=true`). One or several operations can be performed on this row with `row_addmul`, then `row_op_end` must be called. Do not use if `inverse_transform_enabled=true`.

**d**

Number of rows of *B* (dimension of the lattice).

```
>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(11, 11)
>>> M = GSO.Mat(A)
>>> M.d
11
```

**discover\_all\_rows** (*self*)

Allows `row_addmul` for all rows even if the GSO has never been computed.

**float\_type**

```
>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(10, 10)
>>> M = GSO.Mat(A)
>>> M.float_type
'double'
>>> set_precision(100)
53
>>> M = GSO.Mat(A, float_type='mpfr')
>>> M.float_type
'mpfr'
```

**from\_canonical** (*self*, *v*, *int start=0*, *int dimension=-1*)

Given a vector  $v$  wrt the canonical basis  $\mathbb{Z}^n$  return a vector wrt the Gram-Schmidt basis  $B^*$

#### Parameters

- **v** – a tuple-like object of dimension `M.B.ncols`
- **start** – only consider subbasis starting at `start``
- **dimension** – only consider dimension vectors or all if `-1`

**Returns** a tuple of dimension `dimension`` or `M.d`` when `dimension` is `None`

This operation is the inverse of `to_canonical`:

```
>>> import random
>>> A = IntegerMatrix.random(5, "uniform", bits=6)
>>> M = GSO.Mat(A)
>>> _ = M.update_gso()
>>> v = tuple(IntegerMatrix.random(5, "uniform", bits=6)[0]); v
(35, 24, 55, 40, 23)
>>> w = M.from_canonical(v); w
(0.98294..., 0.5636..., -3.4594479..., 0.9768..., 0.261316...)
>>> v_ = tuple([int(round(wi)) for wi in M.to_canonical(w)]); v_
(35, 24, 55, 40, 23)
>>> v == v_
True
```

**get\_current\_slope** (*self*, *int start\_row*, *int stop\_row*)

Finds the slope of the curve fitted to the lengths of the vectors from `start_row` to `stop_row`. The slope gives an indication of the quality of the LLL-reduced basis.

#### Parameters

- **start\_row** (*int*) – start row index
- **stop\_row** (*int*) – stop row index (exclusive)

---

**Note:** we call `get_current_slope` which is declared in `bkz.h`

---

**get\_gram** (*self*, *int i*, *int j*)

Return Gram matrix coefficients ( $0 \leq i < n\_known\_rows$  and  $0 \leq j < i$ ). If `enable_row_expo` is false, returns the dot product  $b_i, b_j$ . If `enable_row_expo` is true, returns  $b_i, b_j / 2^{(r_i+r_j)}$ , where  $r_i$  and  $r_j$  are the row exponents of rows  $i$  and  $j$  respectively.

#### Parameters

- **i** (*int*) –
- **j** (*int*) –

**get\_log\_det** (*self*, *int start\_row*, *int stop\_row*)

Return log of the (squared) determinant of the basis.

#### Parameters

- **start\_row** (*int*) – start row (inclusive)
- **stop\_row** (*int*) – stop row (exclusive)

**get\_mu** (*self*, *int i*, *int j*)

Return  $\langle b_i, b_j^* \rangle / \|b_j^*\|^2$ .

**Parameters**

- **i** –
- **j** –

**get\_mu\_exp** (*self*, *int i*, *int j*)

Return  $f =_{i,j}$  and exponent  $x$  such that  $f2^x = b_i, b_j^*/b_j^{*2}$ . If `enable_row_expo` is false,  $x$  is always zero. If `enable_row_expo` is true,  $x = r_i - r_j$ , where  $r_i$  and  $r_j$  are the row exponents of rows  $i$  and  $j$  respectively.

---

**Note:** It is assumed that  $_{i,j}$  is valid.

---

**Parameters**

- **i** –
- **j** –

**get\_r** (*self*, *int i*, *int j*)

Return  $b_i, b_j^*$ .

**Parameters**

- **i** –
- **j** –

```
>>> from fpylll import *
>>> A = IntegerMatrix.random(5, "uniform", bits=5)
>>> M = GSO.Mat(A)
>>> M.update_gso()
True
>>> M.get_r(1, 0)
833.0
```

**get\_r\_exp** (*self*, *int i*, *int j*)

Return  $f = r_{i,j}$  and exponent  $x$  such that  $b_i, b_j^* = f2^x$ . If `enable_row_expo` is false,  $x$  is always 0. If `enable_row_expo` is true,  $x = r_i + r_j$ , where  $r_i$  and  $r_j$  are the row exponents of rows  $i$  and  $j$  respectively.

---

**Note:** It is assumed that  $r(i, j)$  is valid.

---

**Parameters**

- **i** –
- **j** –

**get\_root\_det** (*self*, *int start\_row*, *int stop\_row*)

Return (squared) root determinant of the basis.

**Parameters**

- **start\_row** (*int*) – start row (inclusive)
- **stop\_row** (*int*) – stop row (exclusive)

**get\_slide\_potential** (*self*, *int start\_row*, *int stop\_row*, *int block\_size*)

Return slide potential of the basis

**Parameters**

- **start\_row** (*int*) – start row (inclusive)
- **stop\_row** (*int*) – stop row (exclusive)
- **block\_size** (*int*) – block size

**int\_gram\_enabled**

Exact computation of dot products.

```
>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(11, 11)
>>> M = GSO.Mat(A)
>>> M.int_gram_enabled
False
```

```
>>> M = GSO.Mat(A, flags=GSO.INT_GRAM)
>>> M.int_gram_enabled
True
```

**inverse\_transform\_enabled**

Computation of the inverse transform matrix (transposed).

```
>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(11, 11)
>>> M = GSO.Mat(A)
>>> M.inverse_transform_enabled
False
```

```
>>> U = IntegerMatrix.identity(11)
>>> UinvT = IntegerMatrix.identity(11)
>>> M = GSO.Mat(A, U=U, UinvT=UinvT)
>>> M.inverse_transform_enabled
True
```

**move\_row** (*self*, *int old\_r*, *int new\_r*)

Row *old\_r* becomes row *new\_r* and intermediate rows are shifted. If *new\_r* < *old\_r*, then *old\_r* must be < *n\_known\_rows*.

**Parameters**

- **old\_r** (*int*) – row index
- **new\_r** (*int*) – row index

**negate\_row** (*self*, *int i*)

Set  $b_i$  to  $-b_i$ .

**Parameters** **i** (*int*) – index of the row to negate

Example:

```
>>> from fpylll import *
>>> set_random_seed(42)
>>> A = IntegerMatrix(6, 6)
>>> A.randomize("ntrulike", bits=6, q=31)
>>> print(A)
```

```

[ 1 0 0 12 25 25 ]
[ 0 1 0 25 12 25 ]
[ 0 0 1 25 25 12 ]
[ 0 0 0 31 0 0 ]
[ 0 0 0 0 31 0 ]
[ 0 0 0 0 0 31 ]

>>> M = GSO.Mat(A)
>>> M.update_gso()
True
>>> with M.row_ops(2,2):
...     M.negate_row(2)
...
>>> print(A)
[ 1 0 0 12 25 25 ]
[ 0 1 0 25 12 25 ]
[ 0 0 -1 -25 -25 -12 ]
[ 0 0 0 31 0 0 ]
[ 0 0 0 0 31 0 ]
[ 0 0 0 0 0 31 ]

```

**r** (*self*, *start=0*, *end=-1*)

Return *r* vector from *start* to *end*

**remove\_last\_row** (*self*)

Remove. the last row of *B* (and of *U* if *enable\_transform=true*). Do not use if *inverse\_transform\_enabled=true*.

**row\_addmul** (*self*, *int i*, *int j*, *x*)

Set  $b_i = b_i + xb_j$ .

After one or several calls to *row\_addmul*, *row\_op\_end* must be called.

If *row\_op\_force\_long=true*, *x* is always converted to  $(2^{\text{expo}} * \text{long})$  instead of  $(2^{\text{expo}} * \text{ZT})$ , which is faster if *ZT=mpz\_t* but might lead to a loss of precision in LLL, more Babai iterations are needed.

#### Parameters

- *i* (*int*) – target row
- *j* (*int*) – source row
- *x* – multiplier

**row\_expo\_enabled**

Normalization of each row of *b* by a power of 2.

```

>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(11, 11)
>>> M = GSO.Mat(A)
>>> M.row_expo_enabled
False

```

```

>>> M = GSO.Mat(A, flags=GSO.ROW_EXPO)
>>> M.row_expo_enabled
True

```

**row\_op\_begin** (*self*, *int first*, *int last*)

Must be called before a sequence of *row\_addmul*.

**Parameters**

- **first** (*int*) – start index for row\_addmul operations.
- **last** (*int*) – final index (exclusive).

---

**Note:** It is preferable to use `MatGSORowOpContext` via `row_ops`.

---

**row\_op\_end** (*self, int first, int last*)

Must be called after a sequence of `row_addmul`. This invalidates the *i*-th line of the GSO.

**Parameters**

- **first** (*int*) – start index to invalidate.
- **last** (*int*) – final index to invalidate (exclusive).

---

**Note:** It is preferable to use `MatGSORowOpContext` via `row_ops`.

---

**row\_op\_force\_long**

Changes the behaviour of `row_addmul`, see its documentation.

```
>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(11, 11)
>>> M = GSO.Mat(A)
>>> M.row_op_force_long
False
```

```
>>> M = GSO.Mat(A, flags=GSO.OP_FORCE_LONG)
>>> M.row_op_force_long
True
```

**row\_ops** (*self, int first, int last*)

Return context in which `row_addmul` operations are safe.

**Parameters**

- **first** (*int*) – start index.
- **last** (*int*) – final index (exclusive).

**to\_canonical** (*self, v, int start=0*)

Given a vector *v* wrt the Gram-Schmidt basis  $B^*$  return a vector wrt the canonical basis  $\mathbb{Z}^n$

**Parameters**

- **v** – a tuple-like object of dimension `M.d`
- **start** – only consider subbasis starting at `start``

**Returns** a tuple of dimension `M.B.ncols`

**transform\_enabled**

Computation of the transform matrix.

```
>>> from fpylll import IntegerMatrix, GSO, set_precision
>>> A = IntegerMatrix(11, 11)
>>> M = GSO.Mat(A)
>>> M.transform_enabled
False
```

```
>>> U = IntegerMatrix.identity(11)
>>> M = GSO.Mat(A, U=U)
```

```
>>> M.transform_enabled
True
```

**update\_gso** (*self*)

Updates all GSO coefficients ( and  $r$ ).

**update\_gso\_row** (*self*, *int i*, *int last\_j*)

Updates  $r_{i,j}$  and  $i_{i,j}$  if needed for all  $j$  in  $[0, last_j]$ . All coefficients of  $r$  and above the  $i$ -th row in columns  $[0, \min(last_j, i - 1)]$  must be valid.

**Parameters**

- **i** (*int*) –
- **last\_j** (*int*) –

**class** fpylll.fplll.gso.**MatGSORowOpContext** (*M*, *i*, *j*)

A context in which performing row operations is safe. When the context is left, the appropriate updates are performed by calling `row_op_end()`.

**\_\_init\_\_** (*self*, *M*, *i*, *j*)

Construct new context for  $M[i:j]$ .

**Parameters**

- **M** – MatGSO object
- **i** – start row
- **j** – stop row

## LLL Wrapper

**class** fpylll.fplll.wrapper.**Wrapper** (*IntegerMatrix B*, *double delta=LLL\_DEF\_DELTA*, *double eta=LLL\_DEF\_ETA*, *int flags=LLL\_DEFAULT*)**B****U****UinvT****\_\_call\_\_**

Run LLL.

**Returns****Return type**

```
>>> from fpylll import LLL, IntegerMatrix, GSO
>>> A = IntegerMatrix(40, 40)
>>> A.randomize("ntrulike", bits=10, q=1023)
>>> W = LLL.Wrapper(A)
>>> W()
```

**\_\_init\_\_**

FIXME! briefly describe function

### Parameters

- **B** (*IntegerMatrix*) –
- **delta** (*double*) –
- **eta** (*double*) –
- **flags** (*int*) –

```
>>> from fpylll import LLL, IntegerMatrix
>>> A = IntegerMatrix(50, 50)
>>> A.randomize("ntrulike", bits=100, q=1023)
>>> W = LLL.Wrapper(A)
```

**status**

## LLL

class fpylll.fplll.lll.LLL

**DEFAULT = 0**

**DEFAULT\_DELTA = 0.99**

**DEFAULT\_ETA = 0.51**

**EARLY\_RED = 2**

**Reduction**

alias of *LLLReduction*

**SIEGEL = 4**

**VERBOSE = 1**

class **Wrapper** (*IntegerMatrix B, double delta=LLL\_DEF\_DELTA, double eta=LLL\_DEF\_ETA, int flags=LLL\_DEFAULT*)

**B**

**U**

**UinvT**

**\_\_call\_\_**

Run LLL.

**Returns**

**Return type**

```
>>> from fpylll import LLL, IntegerMatrix, GSO
>>> A = IntegerMatrix(40, 40)
>>> A.randomize("ntrulike", bits=10, q=1023)
>>> W = LLL.Wrapper(A)
>>> W()
```

**\_\_init\_\_**

FIXME! briefly describe function

**Parameters**

- **B** (*IntegerMatrix*) –
- **delta** (*double*) –

- **eta** (*double*) –
- **flags** (*int*) –

```
>>> from fpylll import LLL, IntegerMatrix
>>> A = IntegerMatrix(50, 50)
>>> A.randomize("ntrulike", bits=100, q=1023)
>>> W = LLL.Wrapper(A)
```

#### status

**static is\_reduced** (*M*, *delta*=0.99, *eta*=0.51)

`is_LLL_reduced(M, delta=LLL_DEF_DELTA, eta=LLL_DEF_ETA)` Test if *M* is LLL reduced.

**param M** either an GSO object of an integer matrix or an integer matrix.

**param delta** LLL parameter  $\delta < 1$

**param eta** LLL parameter  $\eta > 0.5$

**returns** Return True if *M* is definitely LLL reduced, False otherwise.

Random matrices are typically not LLL reduced:

```
>>> from fpylll import IntegerMatrix, LLL
>>> A = IntegerMatrix(40, 40)
>>> A.randomize('uniform', bits=32)
>>> LLL.is_reduced(A)
False
```

LLL reduction should produce matrices which are LLL reduced:

```
>>> LLL.reduction(A)
<IntegerMatrix(40, 40) at 0x...>
>>> LLL.is_reduced(A)
True
```

---

**Note:** This function may return False for LLL reduced matrices if the precision used to compute the GSO is too small.

---

**static reduction** (*B*, *U*=None, *delta*=0.99, *eta*=0.51, *method*=None, *float\_type*=None, *precision*=0, *flags*=0)

`lll_reduction(IntegerMatrix B, U=None, double delta=LLL_DEF_DELTA, double eta=LLL_DEF_ETA, method=None, float_type=None, int precision=0, int flags=LLL_DEFAULT)` Run LLL reduction.

**param IntegerMatrix B** Integer matrix, modified in place.

**param U** Transformation matrix or None

**param double delta** LLL parameter  $0.25 < 1$

**param double eta** LLL parameter  $0 <$

**param method** one of 'wrapper', 'proved', 'heuristic', 'fast' or None.

**param float\_type** an element of `fpylll.float_types` or None

**param precision** bit precision to use if `float_tpe` is 'mpfr'

**param int flags** LLL flags.

**returns** modified matrix *B*

```
class fpylll.fpylll.lll.LLLReduction(MatGSO M, double delta=LLL_DEF_DELTA, double
eta=LLL_DEF_ETA, int flags=LLL_DEFAULT)
```

**M**

**\_\_call\_\_**

LLL reduction.

**Parameters**

- **kappa\_min** (*int*) – minimal index to go back to
- **kappa\_start** (*int*) – index to start processing at
- **kappa\_end** (*int*) – end index (exclusive)
- **size\_reduction\_start** (*int*) – only perform size reductions using vectors starting at this index

**\_\_init\_\_**

FIXME! briefly describe function

**Parameters**

- **M** (*MatGSO*) –
- **delta** (*double*) –
- **eta** (*double*) –
- **flags** (*int*) –
  - DEFAULT:
  - VERBOSE:
  - EARLY\_RED:
  - SIEGEL:

**delta**

**eta**

**final\_kappa**

FIXME! briefly describe function

**Returns**

**Return type**

**last\_early\_red**

FIXME! briefly describe function

**Returns**

**Return type**

**nswaps**

FIXME! briefly describe function

**Returns**

**Return type**

**size\_reduction** (*self*, *int kappa\_min=0*, *int kappa\_end=-1*, *int size\_reduction\_start=0*)

Size reduction.

**Parameters**

- **kappa\_min** (*int*) – start index
- **kappa\_end** (*int*) – end index (exclusive)
- **size\_reduction\_start** (*int*) – only perform size reductions using vectors starting at this index

**zeros**

FIXME! briefly describe function

**Returns****Return type**

`fpylll.fplll.lll.is_LLL_reduced` (*M*, *delta*=`LLL_DEF_DELTA`, *eta*=`LLL_DEF_ETA`)

Test if *M* is LLL reduced.

**Parameters**

- **M** – either an GSO object of an integer matrix or an integer matrix.
- **delta** – LLL parameter  $\delta < 1$
- **eta** – LLL parameter  $\eta > 0.5$

**Returns** Return `True` if *M* is definitely LLL reduced, `False` otherwise.

Random matrices are typically not LLL reduced:

```
>>> from fpylll import IntegerMatrix, LLL
>>> A = IntegerMatrix(40, 40)
>>> A.randomize('uniform', bits=32)
>>> LLL.is_reduced(A)
False
```

LLL reduction should produce matrices which are LLL reduced:

```
>>> LLL.reduction(A)
<IntegerMatrix(40, 40) at 0x...>
>>> LLL.is_reduced(A)
True
```

---

**Note:** This function may return `False` for LLL reduced matrices if the precision used to compute the GSO is too small.

---

`fpylll.fplll.lll.lll_reduction` (*IntegerMatrix* *B*, *U*=`None`, *double* *delta*=`LLL_DEF_DELTA`, *double* *eta*=`LLL_DEF_ETA`, *method*=`None`, *float\_type*=`None`, *int* *precision*=`0`, *int* *flags*=`LLL_DEFAULT`)

Run LLL reduction.

**Parameters**

- **B** (*IntegerMatrix*) – Integer matrix, modified in place.
- **U** – Transformation matrix or `None`
- **delta** (*double*) – LLL parameter  $0.25 < 1$
- **eta** (*double*) – LLL parameter  $0 <$
- **method** – one of 'wrapper', 'proved', 'heuristic', 'fast' or `None`.
- **float\_type** – an element of `fpylll.float_types` or `None`

- **precision** – bit precision to use if `float_tpe` is 'mpfr'
- **flags** (*int*) – LLL flags.

**Returns** modified matrix B

## BKZ

Block Korkine Zolotarev algorithm.

`class fpylll.fplll.bkz.BKZ`

**AUTO\_ABORT = 32**

**AutoAbort**

alias of *BKZAutoAbort*

**BOUNDED\_LLL = 16**

**DEFAULT = 0**

**DEFAULT\_AUTO\_ABORT\_MAX\_NO\_DEC = 5**

**DEFAULT\_AUTO\_ABORT\_SCALE = 1.0**

**DEFAULT\_GH\_FACTOR = 1.1**

**DEFAULT\_MIN\_SUCCESS\_PROBABILITY = 0.5**

**DEFAULT\_RERANDOMIZATION\_DENSITY = 3**

**DEFAULT\_STRATEGY = '/home/docs/checkouts/readthedocs.org/user\_builds/fpylll/conda/latest/share/fplll/strategies/default'**

**DEFAULT\_STRATEGY\_PATH = '/home/docs/checkouts/readthedocs.org/user\_builds/fpylll/conda/latest/share/fplll/strategies/default'**

**DUMP\_GSO = 64**

**GH\_BND = 128**

**MAX\_LOOPS = 4**

**MAX\_TIME = 8**

**NO\_LLL = 2**

**Param**

alias of *BKZParam*

**Reduction**

alias of *BKZReduction*

**SD\_VARIANT = 256**

**SLD\_RED = 512**

**VERBOSE = 1**

**static reduction** (*B*, *o*, *float\_type=None*, *precision=0*)

`bkz_reduction(IntegerMatrix B, BKZParam o, float_type=None, int precision=0)`

Run BKZ reduction.

**Parameters**

- **B** (*IntegerMatrix*) – Integer matrix, modified in place.
- **o** (*BKZParam*) – BKZ parameters

- **float\_type** – either None: for automatic choice or an entry of *fpyl111.float\_types*
- **precision** – bit precision to use if `float_tpe` is 'mpfr'

**Returns** modified matrix B

**class** `fpyl111.fpl111.bkz.BKZAutoAbort` (*MatGSO M, int num\_rows, int start\_row=0*)  
 Utility class for aborting BKZ when slope does not improve any longer.

**\_\_init\_\_**

Create new auto abort object.

**Parameters**

- **M** – GSO matrix
- **num\_rows** – number of rows
- **start\_row** – start at this row

**test\_abort** (*self, scale=1.0, int max\_no\_dec=5*)

Test if new slope fails to be smaller than  $scale * old\_slope$  for  $max\_no\_dec$  iterations.

**Parameters**

- **scale** – target decrease
- **max\_no\_dec** (*int*) – number of rounds allowed to be stuck

**class** `fpyl111.fpl111.bkz.BKZReduction` (*MatGSO M, LLLReduction lll\_obj, BKZParam param*)

**M**

**\_\_call\_\_**

Call BKZ, SD-BKZ or slide reduction.

---

**Note:** To enable the latter, set flags `BKZ.SLD_RED` or `BKZ.SD_VARIANT` when calling the constructor of this class.

---

**\_\_init\_\_**

Construct new BKZ object.

**Parameters**

- **M** – GSO object
- **lll\_obj** – LLL object called as a subroutine
- **param** – parameters

**dsvp\_postprocessing** (*self, int kappa, int block\_size, tuple solution*)

Insert solution into basis after Dual-SVP oracle call

**Parameters**

- **kappa** – index
- **block\_size** – block size
- **solution** – solution to insert

**hkz** (*self, BKZParam param, int min\_row, int max\_row*)

HKZ reduction between `min_row` and `max_row`.

**Parameters**

- **param** – reduction parameters
- **min\_row** – start row
- **max\_row** – maximum row to consider (exclusive)

**Returns** True if no changes were made, False otherwise.

**lll\_obj**

**nodes**

Total number of enumeration nodes visited during this reduction.

**param**

**rerandomize\_block** (*self, int min\_row, int max\_row, int density*)

Rerandomize block between *min\_row* and *max\_row* with a transform of *density*

**Parameters**

- **min\_row** –
- **max\_row** –
- **density** –

**sd\_tour** (*self, int loop, BKZParam param, int min\_row, int max\_row*)

One Dual-BKZ tour.

**Parameters**

- **loop** – loop index
- **param** – reduction parameters
- **min\_row** – start row
- **max\_row** – maximum row to consider (exclusive)

**Returns** True if no changes were made, False otherwise.

**slide\_tour** (*self, int loop, BKZParam param, int min\_row, int max\_row*)

One slide reduction tour.

**Parameters**

- **loop** – loop index
- **param** – reduction parameters
- **min\_row** – start row
- **max\_row** – maximum row to consider (exclusive)

**Returns** True if no changes were made, False otherwise.

---

**Note:** You must run `lll_obj()` before calling this function, otherwise this function will produce an error.

---

**status**

Status of this reduction.

**svp\_postprocessing** (*self, int kappa, int block\_size, tuple solution*)

Insert solution into basis after SVP oracle call

**Parameters**

- **kappa** – index
- **block\_size** – block size
- **solution** – solution to insert

**svp\_preprocessing** (*self, int kappa, int block\_size, BKZParam param*)

Preprocess before calling (Dual-)SVP oracle.

**Parameters**

- **kappa** – index
- **block\_size** – block size
- **param** – reduction parameters

**svp\_reduction** (*self, int kappa, int block\_size, BKZParam param, dual=False*)

Run (Dual-)SVP reduction (incl. pre and postprocessing)

**Parameters**

- **kappa** – index
- **block\_size** – block size
- **param** – reduction parameters
- **dual** – dual or primal reduction

**tour** (*self, int loop, BKZParam param, int min\_row, int max\_row*)

One BKZ tour.

**Parameters**

- **loop** – loop index
- **param** – reduction parameters
- **min\_row** – start row
- **max\_row** – maximum row to consider (exclusive)

**Returns** tuple (*clean, max\_kappa*) where *clean* == True if no changes were made, and *max\_kappa* is the maximum index for which no changes were made.

`fpyl111.fpl111.bkz.bkz_reduction` (*IntegerMatrix B, BKZParam o, float\_type=None, int precision=0*)

Run BKZ reduction.

**Parameters**

- **B** (*IntegerMatrix*) – Integer matrix, modified in place.
- **o** (*BKZParam*) – BKZ parameters
- **float\_type** – either None: for automatic choice or an entry of *fpyl111.float\_types*
- **precision** – bit precision to use if *float\_tpe* is 'mpfr'

**Returns** modified matrix B

## SVP and CVP

Shortest and Closest Vectors.

class fpylll.fplll.svpcvp.CVP

**DEFAULT = 0**

**VERBOSE = 1**

**static** `closest_vector` (*IntegerMatrix B, target, int flags=CVP\_DEFAULT*)

Return a closest vector.

**Parameters**

- **B** (*IntegerMatrix*) –
- **target** (*vector [Z\_NR [mpz\_t]]*) –
- **flags** (*int*) –

**Returns coordinates of the solution vector**

**Rtype tuple**

```
>>> from fpylll import *
>>> set_random_seed(42)
>>> A = IntegerMatrix.random(5, 'uniform', bits=10)
>>> lll = LLL.reduction(A)
>>> t = (94, -42, 123, 512, -1337)
>>> print (CVP.closest_vector(A, t))
(-34, 109, 204, 360, -1548)
```

class fpylll.fplll.svpcvp.SVP

**DEFAULT = 0**

**OVERRIDE\_BND = 2**

**VERBOSE = 1**

**static** `shortest_vector` (*IntegerMatrix B, method=None, int flags=SVP\_DEFAULT, pruning=None,*

*run\_lll=True, max\_aux\_sols=0*)

Return a shortest vector.

**Parameters**

- **B** (*IntegerMatrix*) –
- **method** –
- **flags** (*int*) –
- **pruning** –
- **run\_lll** –
- **max\_aux\_sols** –

**Returns**

**Return type**

fpylll.fplll.svpcvp.`closest_vector` (*IntegerMatrix B, target, int flags=CVP\_DEFAULT*)

Return a closest vector.

**Parameters**

- **B** (*IntegerMatrix*) –

- **target** (*vector*[*Z\_NR*[*mpz\_t*]]) –
- **flags** (*int*) –

Returns coordinates of the solution vector

Rtype tuple

```
>>> from fpylll import *
>>> set_random_seed(42)
>>> A = IntegerMatrix.random(5, 'uniform', bits=10)
>>> lll = LLL.reduction(A)
>>> t = (94, -42, 123, 512, -1337)
>>> print (CVP.closest_vector(A, t))
(-34, 109, 204, 360, -1548)
```

`fpylll.fpylll.svpcvp.shortest_vector` (*IntegerMatrix* *B*, *method=None*, *int* *flags=SVP\_DEFAULT*, *pruning=None*, *run\_lll=True*, *max\_aux\_sols=0*)

Return a shortest vector.

Parameters

- **B** (*IntegerMatrix*) –
- **method** –
- **flags** (*int*) –
- **pruning** –
- **run\_lll** –
- **max\_aux\_sols** –

Returns

Return type

## Enumeration

`class fpylll.fpylll.enumeration.Enumeration` (*MatGSO* *M*, *nr\_solutions=1*, *strategy=EvaluatorStrategy.BEST\_N\_SOLUTIONS*)

**M**

`__init__`

Create new enumeration object

Parameters **M** (*MatGSO*) – GSO matrix

`enumerate` (*self*, *int first*, *int last*, *max\_dist*, *max\_dist\_expo*, *target=None*, *subtree=None*, *pruning=None*, *dual=False*, *subtree\_reset=False*)

Run enumeration on *M*

Parameters

- **first** (*int*) – first row
- **last** (*int*) – last row (exclusive)
- **max\_dist** – length bound
- **max\_dist\_expo** – exponent of length bound

- **target** – target coordinates for CVP/BDD or None for SVP
- **subtree** –
- **pruning** – pruning parameters
- **dual** – run enumeration in the primal or dual lattice.
- **subtree\_reset** –

**Returns** list of pairs containing the solutions and their lengths

**get\_nodes** (*self*)

Return number of visited nodes in last enumeration call.

**exception** `fpyll.fplll.enumeration.EnumerationError`

**class** `fpyll.fplll.enumeration.EvaluatorStrategy`

**BEST\_N\_SOLUTIONS** = 0

**FIRST\_N\_SOLUTIONS** = 2

**OPPORTUNISTIC\_N\_SOLUTIONS** = 1

## Utilities

**class** `fpyll.util.PrecisionContext` (*prec*)

**\_\_init\_\_** (*self*, *prec*)

Create new precision context.

**Parameters** *prec* – internal precision

**exception** `fpyll.util.ReductionError`

`fpyll.util.adjust_radius_to_gh_bound` (*double dist*, *int dist\_expo*, *int block\_size*, *double root\_det*, *double gh\_factor*)

Use Gaussian Heuristic to reduce bound on the length of the shortest vector.

**Parameters**

- **dist** (*double*) – norm of shortest vector
- **dist\_expo** (*int*) – exponent of norm (for dpe representation)
- **block\_size** (*int*) – block size
- **root\_det** (*double*) – root determinant
- **gh\_factor** (*double*) – factor to multiply with

**Returns** (*dist*, *expo*)

`fpyll.util.ball_log_vol` (*n*)

`fpyll.util.gaussian_heuristic` (*r*)

`fpyll.util.get_precision` (*float\_type*='mpfr')

Get currently set precision

**Parameters** *float\_type* – one of 'double', 'long double', 'dpe', 'dd', 'qd' or 'mpfr'

**Returns** precision in bits

This function returns the precision per type:

```
>>> from fpylll import get_precision, set_precision
>>> get_precision('double')
53
>>> get_precision('long double')
64
>>> get_precision('dpe')
53
```

For the MPFR type different precisions are supported:

```
>>> _ = set_precision(212)
>>> get_precision('mpfr')
212
>>> get_precision()
212
>>> _ = set_precision(53)
```

`fpylll.util.precision` (*prec*)

Create new precision context.

**Parameters** `prec` – internal precision

`fpylll.util.set_precision` (*unsigned int prec*)

Set precision globally for MPFR

**Parameters** `prec` – an integer  $\geq 53$

**Returns** current precision

`fpylll.util.set_random_seed` (*unsigned long seed*)

Set random seed.

**Parameters** `seed` – a new seed.

## Python Algorithms

The modules in this category extend the functionality of fpylll in some way by implementing algorithms in Python.

### Simple BKZ

A minimal implementation of the Block Korkine Zolotarev algorithm in Python.

```
class fpylll.algorithms.simple_bkz.BKZReduction(A)
```

```
    __call__(block_size)
```

Perform BKZ reduction with given “`block_size`”.

Nothing is returned, the matrix `A` given during construction is modified in-place.

**Parameters** `block_size` – an integer  $> 2$

```
    __init__(A)
```

Construct a new BKZ reduction instance.

**Parameters** `A` – Integer matrix to reduce.

**bkz\_loop** (*block\_size, min\_row, max\_row*)

Perform one BKZ loop, often also called a “BKZ tour”.

**Parameters**

- **block\_size** – an integer > 2
- **min\_row** – algorithm starts in this row (inclusive)
- **max\_row** – algorithm stops at this row (exclusive)

**svp\_reduction** (*kappa, block\_size*)

Call the SVP oracle and insert found vector into basis.

**Parameters**

- **kappa** – row index
- **block\_size** – an integer > 2

## Simple Dual BKZ

`class fpylll.algorithms.simple_dbkz.DBKZReduction(A)`

**bkz\_loop** (*block\_size, min\_row, max\_row*)

FIXME! briefly describe function

**Parameters**

- **block\_size** –
- **min\_row** –
- **max\_row** –

**Returns**

**Return type**

**dsvp\_reduction** (*kappa, block\_size*)

FIXME! briefly describe function

**Parameters**

- **kappa** –
- **block\_size** –

**Returns**

**Return type**

**euclid** (*pair1, pair2*)

FIXME! briefly describe function

**Parameters**

- **pair1** –
- **pair2** –

**Returns**

**Return type**

## BKZ

Block Korkine Zolotarev algorithm in Python.

This module reimplements fpylll's BKZ algorithm in Python. It has feature parity with the C++ implementation in fpylll's core. Additionally, this implementation collects some additional statistics. Hence, it should provide a good basis for implementing variants of this algorithm.

**class** `fpylll.algorithms.bkz.BKZReduction` (*A*)

An implementation of the BKZ algorithm in Python.

This class has feature parity with the C++ implementation in fpylll's core. Additionally, this implementation collects some additional statistics. Hence, it should provide a good basis for implementing variants of this algorithm.

`__call__` (*params*, *min\_row=0*, *max\_row=-1*)

Run the BKZ algorithm with parameters *param*.

### Parameters

- **params** – BKZ parameters
- **min\_row** – start processing in this row
- **max\_row** – stop processing in this row (exclusive)

`__init__` (*A*)

Construct a new instance of the BKZ algorithm.

**Parameters** **A** – an integer matrix, a GSO object or an LLL object

**svp\_call** (*kappa*, *block\_size*, *params*, *tracer=None*)

Call SVP oracle

### Parameters

- **kappa** – current index
- **params** – BKZ parameters
- **block\_size** – block size
- **tracer** – object for maintaining statistics

**Returns** Coordinates of SVP solution or `None` if none was found.

---

**Note:** `block_size` may be smaller than `params.block_size` for the last blocks.

---

**svp\_postprocessing** (*kappa*, *block\_size*, *solution*, *tracer*)

Insert SVP solution into basis and LLL reduce.

### Parameters

- **solution** – coordinates of an SVP solution
- **kappa** – current index
- **block\_size** – block size
- **tracer** – object for maintaining statistics

**Returns** `True` if no change was made and `False` otherwise

**svp\_preprocessing** (*kappa*, *block\_size*, *params*, *tracer*)

Perform preprocessing for calling the SVP oracle

**Parameters**

- **kappa** – current index
- **params** – BKZ parameters
- **block\_size** – block size
- **tracer** – object for maintaining statistics

**Returns** `True` if no change was made and `False` otherwise

---

**Note:** `block_size` may be smaller than `params.block_size` for the last blocks.

---

**svp\_reduction** (*kappa*, *block\_size*, *params*, *tracer*=<fpylll.algorithms.bkz\_stats.Tracer object>)

Find shortest vector in projected lattice of dimension `block_size` and insert into current basis.

**Parameters**

- **kappa** – current index
- **params** – BKZ parameters
- **block\_size** – block size
- **tracer** – object for maintaining statistics

**Returns** `True` if no change was made and `False` otherwise

**tour** (*params*, *min\_row*=0, *max\_row*=-1, *tracer*=<fpylll.algorithms.bkz\_stats.Tracer object>)

One BKZ loop over all indices.

**Parameters**

- **params** – BKZ parameters
- **min\_row** – start index 0
- **max\_row** – last index n

**Returns** `True` if no change was made and `False` otherwise



## CHAPTER 3

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