
AMGCL Documentation

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AMGCL is a header-only C++ library for solving large sparse linear systems with algebraic multigrid (AMG) method. AMG is one of the most effective iterative methods for solution of equation systems arising, for example, from discretizing PDEs on unstructured grids [Stue99], [TrOS01]. The method can be used as a black-box solver for various computational problems, since it does not require any information about the underlying geometry. AMG is often used not as a standalone solver but as a preconditioner within an iterative solver (e.g. Conjugate Gradients, BiCGStab, or GMRES).

AMGCL builds the AMG hierarchy on a CPU and then transfers it to one of the provided backends. This allows for transparent acceleration of the solution phase with help of OpenCL, CUDA, or OpenMP technologies. Users may provide their own backends which enables tight integration between AMGCL and the user code.

The library source code is available under MIT license at <https://github.com/ddemidov/amgcl>.

Getting started

The easiest way to solve a problem with AMGCL is to use the `amgcl::make_solver` class. It has two template parameters: the first one specifies a *preconditioner* to use, and the second chooses an *iterative solver*. The class constructor takes the system matrix in one of supported *formats* and parameters for the chosen algorithms and for the *backend*.

Solving Poisson's equation

Let us consider a simple example of *Poisson's equation* in a unit square. Here is how the problem may be solved with AMGCL. We will use BiCGStab solver preconditioned with smoothed aggregation multigrid with SPAI(0) for relaxation (smoothing). First, we include the necessary headers. Each of those brings in the corresponding component of the method:

```
#include <amgcl/make_solver.hpp>
#include <amgcl/solver/bicgstab.hpp>
#include <amgcl/amg.hpp>
#include <amgcl/coarsening/smoothed_aggregation.hpp>
#include <amgcl/relaxation/spai0.hpp>
#include <amgcl/adapter/crs_tuple.hpp>
```

Next, we assemble sparse matrix for the Poisson's equation on a uniform 1000x1000 grid. See *Assembling matrix for Poisson's equation* for the source code of the `poisson()` function:

```
std::vector<int> ptr, col;
std::vector<double> val, rhs;
int n = poisson(1000, ptr, col, val, rhs);
```

For this example, we select the *builtin* backend with double precision numbers as value type:

```
typedef amgcl::backend::builtin<double> Backend;
```

Now we can construct the solver for our system matrix. We use the convenient adapter for boost tuples here and just tie together the matrix size and its CRS components:

```
typedef amgcl::make_solver<
    // Use AMG as preconditioner:
    amgcl::amg<
        Backend,
        amgcl::coarsening::smoothed_aggregation,
        amgcl::relaxation::spai0
    >,
    // And BiCGStab as iterative solver:
    amgcl::solver::bicgstab<Backend>
> Solver;

Solver solve( boost::tie(n, ptr, col, val) );
```

Once the solver is constructed, we can apply it to the right-hand side to obtain the solution. This may be repeated multiple times for different right-hand sides. Here we start with a zero initial approximation. The solver returns a boost tuple with number of iterations and norm of the achieved residual:

```
std::vector<double> x(n, 0.0);
int iters;
double error;
boost::tie(iters, error) = solve(rhs, x);
```

That's it! Vector `x` contains the solution of our problem now.

Input formats

We used STL vectors to store the matrix components in the above example. This may seem too restrictive if you want to use AMGCL with your own types. But the `crs_tuple` adapter will take anything that the `Boost.Range` library recognizes as a random access range. For example, you can wrap raw pointers to your data into a `boost::iterator_range`:

```
Solver solve( boost::make_tuple(
    n,
    boost::make_iterator_range(ptr.data(), ptr.data() + ptr.size()),
    boost::make_iterator_range(col.data(), col.data() + col.size()),
    boost::make_iterator_range(val.data(), val.data() + val.size())
) );
```

Same applies to the right-hand side and the solution vectors. And if that is still not general enough, you can provide your own adapter for your matrix type. See *Matrix adapters* for further information on this.

Setting parameters

Any component in AMGCL defines its own parameters by declaring a `param` subtype. When a class wraps several subclasses, it includes parameters of its children into its own `param`. For example, parameters for the `amgcl::make_solver<Precond, Solver>` are declared as

```
struct params {
    typename Precond::params precondition;
    typename Solver::params solver;
};
```


Knowing that, we can easily set the parameters for individual components. For example, we can set the desired tolerance for the iterative solver in the above example like this:

```
Solver::params prm;
prm.solver.tol = 1e-3;
Solver solve( boost::tie(n, ptr, col, val), prm );
```

Parameters may also be initialized with a `boost::property_tree::ptree`. This is especially convenient when *Runtime interface* is used, and the exact structure of the parameters is not known at compile time:

```
boost::property_tree::ptree prm;
prm.put("solver.tol", 1e-3);
Solver solve( boost::tie(n, ptr, col, val), prm );
```

The `make_solver` class

template <class Precond, class IterativeSolver>

class `amgcl::make_solver`

Convenience class that bundles together a preconditioner and an iterative solver.

Public Functions

template <class Matrix>

make_solver (const Matrix &A, const *params* &prm = params (), const backend_params &bprm = backend_params())

Sets up the preconditioner and creates the iterative solver.

template <class Matrix, class Vec1, class Vec2>

boost::tuple<size_t, scalar_type> **operator** () (Matrix const &A, Vec1 const &rhs, Vec2 &&x) const

Computes the solution for the given system matrix A and the right-hand side rhs. Returns the number of iterations made and the achieved residual as a `boost::tuple`. The solution vector x provides initial approximation in input and holds the computed solution on output.

The system matrix may differ from the matrix used during initialization. This may be used for the solution of non-stationary problems with slowly changing coefficients. There is a strong chance that a preconditioner built for a time step will act as a reasonably good preconditioner for several subsequent time steps [DeSh12].

template <class Vec1, class Vec2>

boost::tuple<size_t, scalar_type> **operator** () (Vec1 const &rhs, Vec2 &&x) const

Computes the solution for the given right-hand side rhs. Returns the number of iterations made and the achieved residual as a `boost::tuple`. The solution vector x provides initial approximation in input and holds the computed solution on output.

template <class Vec1, class Vec2>

void **apply** (const Vec1 &rhs, Vec2 &&x) const

Acts as a preconditioner. That is, applies the solver to the right-hand side rhs to get the solution x with zero initial approximation. Iterative methods usually use estimated residual for exit condition. For some problems the value of the estimated residual can get too far from the true residual due to round-off errors. Nesting iterative solvers in this way may allow to shave the last bits off the error. The method should not be used directly but rather allows nesting `make_solver` classes as in the following example:

```
typedef amgcl::make_solver<
    amgcl::make_solver<
        amgcl::amg<
            Backend, amgcl::coarsening::smoothed_aggregation,
            ↪ amgcl::relaxation::spai0
```

```
    >,
    amgcl::solver::cg<Backend>
    >,
    amgcl::solver::cg<Backend>
    > NestedSolver;
```

const Precond &**precond** () **const**

Returns reference to the constructed preconditioner.

const IterativeSolver &**solver** () **const**

Returns reference to the constructed iterative solver.

Precond::matrix **const** &**system_matrix** () **const**

Returns the system matrix in the backend format.

void **get_params** (boost::property_tree::ptree &p) **const**

Stores the parameters used during construction into the property tree p.

size_t **size** () **const**

Returns the size of the system matrix.

struct **params**

Combined parameters of the bundled preconditioner and the iterative solver.

Public Members

template<>

Precond::params **precond**

Preconditioner parameters.

template<>

IterativeSolver::params **solver**

Iterative solver parameters.

Components

Matrix adapters

Matrix adapters in AMGCL allow to construct a solver from some common matrix formats. Internally, the [CRS](#) format is used, but it is easy to adapt any matrix format that allows row-wise access to the nonzero matrix values. An example of creating an adapter is provided in [Adapting custom matrix class](#).

Boost tuple adapter

```
#include <amgcl/adapter/crs_tuple.hpp>
```

The Boost tuple adapter allows to use a `boost::tuple` of a matrix size and its three [CRS](#) format components (row pointer array, column indices array, and values array) as input matrix to AMGCL solvers. The arrays are allowed to be in any format recognized by the [Boost.Range](#) library as a random access range. Common examples are STL vectors and Boost [iterator ranges](#).

Example:

```
// boost::tie creates a tuple of references, which avoids copying.
Solver solve( boost::tie(n, ptr, col, val) );

// A (cheap) copy is required when iterator ranges are created on the fly:
Solver solve( boost::make_tuple(
    n,
    boost::make_iterator_range(ptr.data(), ptr.data() + ptr.size()),
    boost::make_iterator_range(col.data(), col.data() + col.size()),
    boost::make_iterator_range(val.data(), val.data() + val.size())
) );
```

Boost.uBLAS adapter

```
#include <amgcl/adapters/ublas.hpp>
```

The `Boost.uBLAS` adapter allows to use uBLAS sparse matrices as input to AMGCL solvers. It also allows to use uBLAS dense vectors with `amgcl::backend::builtin`.

Example:

```
namespace ublas = boost::numeric::ublas;

ublas::compressed_matrix<double> A;
...
Solver solve(A);

ublas::vector<double> rhs, x;
...
solve(rhs, x);
```

Zero copy adapter

```
#include <amgcl/adapters/zero_copy.hpp>
```

In general, AMGCL copies the adapted input matrix into its internal structures, so that the matrix may be safely destroyed or reused as soon as the solver setup is complete. However, the memory overhead of the copying may be too large, especially for large problems that eat up almost all of available RAM. The zero copy adapter allows to use raw pointers to CRS arrays as input matrix for MAGCL solvers. The data from the arrays is never copied during setup, and the user has to make sure the arrays stay alive long enough. However, unless the backend used is `amgcl::backend::builtin`, the input matrix will be copied into the backend structures when the setup is finished. This would still allow to save some memory in case of GPGPU backends.

The one requirement is that the integer types stored in row pointers and column indices arrays have to be binary compatible with `ptrdiff_t`, and the value type has to be the value type of the backend.

Example:

```
Solver solve( amgcl::adapter::zero_copy(n, &ptr[0], &col[0], &val[0]) );
```

Backends

A backend in AMGCL is a class that defines matrix and vector types together with several operations on them, such as creation, matrix-vector products, elementwise vector operations, inner products etc. The `<amgcl/backend/interface.hpp>` file defines an interface that each backend should extend. The AMG hierarchy is

moved to the specified backend upon construction. The solution phase then uses types and operations defined in the backend. This enables transparent acceleration of the solution phase with OpenMP, OpenCL, CUDA, or any other technologies.

In order to use a backend, user must include its definition from the corresponding file inside `amgcl/backend` folder. On the user side of things, only the types of the right-hand side and the solution vectors should be affected by the choice of AMGCL backend. Here is an example of using the `builtin` backend. First, we need to include the appropriate header:

```
#include <amgcl/backend/builtin.hpp>
```

Then, we need to construct the solver and apply it to the vector types supported by the backend:

```
typedef amgcl::backend::builtin<double> Backend;

typedef amgcl::make_solver<
    amgcl::amg<Backend, amgcl::coarsening::aggregation, amgcl::relaxation::spai0>,
    amgcl::solver::gmres<Backend>
> Solver;

Solver solve(A);

std::vector<double> rhs, x; // Initialized elsewhere

solve(rhs, x);
```

Now, if we want to switch to a different backend, for example, in order to accelerate the solution phase with a powerful GPU, we just need to include another backend header, and change the definitions of `Backend`, `rhs`, and `x`. Here is an example of what needs to be done to use the `VexCL` backend.

Include the correct header:

```
#include <amgcl/backend/builtin.hpp>
```

Change the definition of `Backend`:

```
typedef amgcl::backend::vexcl<double> Backend;
```

Change the definition of the vectors:

```
vex::vector<double> rhs, x;
```

That's it! Well, almost. In case the backend requires some parameters, we also need to provide those. In particular, the `VexCL` backend should know what `VexCL` context to use:

```
// Initialize VexCL context on a single GPU:
vex::Context ctx(vex::Filter::GPU && vex::Filter::Count(1));

// Create backend parameters:
Backend::params backend_prm;
backend_prm.q = ctx;

// Pass the parameters to the solver constructor:
Solver solve(A, Solver::params(), backend_prm);
```

Builtin

```
#include <amgcl/backend/builtin.hpp>
```

```
template <typename ValueType>
```

```
struct amgcl::backend::builtin
```

The builtin backend does not have any dependencies except for the Boost libraries, and uses OpenMP for parallelization. Matrices are stored in the CRS format, and vectors are instances of `std::vector<value_type>`. There is no usual overhead of moving the constructed hierarchy to the builtin backend, since the backend is used internally during setup.

Public Types

```
typedef amgcl::detail::empty_params params
```

The backend has no parameters.

```
struct provides_row_iterator
```

Inherits from `true_type`

VexCL

```
#include <amgcl/backend/vexcl.hpp>
```

```
template <typename real, class DirectSolver = solver::vexcl_skyline_lu<real>>
```

```
struct amgcl::backend::vexcl
```

The backend uses the VexCL library for accelerating solution on the modern GPUs and multicore processors with the help of OpenCL or CUDA technologies. The VexCL backend stores the system matrix as `vex::SpMat<real>` and expects the right hand side and the solution vectors to be instances of the `vex::vector<real>` type.

```
struct params
```

The VexCL backend parameters.

Public Members

```
template<>
```

```
std::vector<vex::backend::command_queue> q
```

Command queues that identify compute devices to use with VexCL.

```
template<>
```

```
bool fast_matrix_setup
```

Do CSR to ELL conversion on the GPU side.

This will result in faster setup, but will require more GPU memory.

```
struct provides_row_iterator
```

Inherits from `false_type`

Preconditioners

Iterative solvers

AMGCL provides several iterative solvers, but it should be easy to use AMGCL preconditioners with a user-provided solver as well. Each solver in AMGCL is a class template. Its single template parameter specifies the backend to use.

This allows to preallocate necessary resources at class construction. Obviously, the solver backend has to coincide with the preconditioner backend.

Each of the solvers in AMGCL provides two overloads for the `operator()`:

```
boost::tuple<size_t, scalar_type> operator () (const Matrix &A, const Precond &P, const Vec1 &rhs, Vec2
&&x) const
```

Computes the solution for the given system matrix `A` and the right-hand side `rhs`. Returns the number of iterations made and the achieved residual as a `boost::tuple`. The solution vector `x` provides initial approximation on input and holds the computed solution on output.

The system matrix may differ from the matrix used during initialization. This may be used for the solution of non-stationary problems with slowly changing coefficients. There is a strong chance that a preconditioner built for a time step will act as a reasonably good preconditioner for several subsequent time steps [DeSh12].

```
boost::tuple<size_t, scalar_type> operator () (const Precond &P, const Vec1 &rhs, Vec2 &&x) const
```

Computes the solution for the given right-hand side `rhs`. The system matrix is the same that was used for the setup of the preconditioner `P`. Returns the number of iterations made and the achieved residual as a `boost::tuple`. The solution vector `x` provides initial approximation on input and holds the computed solution on output.

Conjugate Gradient

```
#include <amgcl/solver/cg.hpp>
```

```
template <class Backend, class InnerProduct = detail::default_inner_product>
```

```
class amgcl::solver::cg
```

Conjugate Gradients method. An effective method for symmetric positive definite systems [Barr94].

Public Functions

```
cg (size_t n, const params &prm = params (), const backend_params &backend_prm = back-
end_params(), const InnerProduct &inner_product = InnerProduct())
Preallocates necessary data structures for the system of size n.
```

```
struct params
```

Solver parameters.

Public Members

```
template<>
```

```
size_t maxiter
```

Maximum number of iterations.

```
template<>
```

```
scalar_type tol
```

Target relative residual error.

```
template<>
```

```
scalar_type abstol
```

Target absolute residual error.

BiConjugate Gradient Stabilized (BiCGSTAB)

```
#include <amgcl/solver/bicgstab.hpp>
```

```
template <class Backend, class InnerProduct = detail::default_inner_product>
```

```
class amgcl::solver::bicgstab
```

BiConjugate Gradient Stabilized (BiCGSTAB) method. The BiConjugate Gradient Stabilized method (BiCGSTAB) was developed to solve nonsymmetric linear systems while avoiding the often irregular convergence patterns of the Conjugate Gradient [Barr94].

Public Functions

```
bicgstab (size_t n, const params &prm = params (), const backend_params &backend_prm = backend_params(), const InnerProduct &inner_product = InnerProduct())
    Preallocates necessary data structures for the system of size n.
```

```
struct params
```

Solver parameters.

Public Members

```
template<>
size_t maxiter
    Maximum number of iterations.
```

```
template<>
scalar_type tol
    Target relative residual error.
```

```
template<>
scalar_type abstol
    Target absolute residual error.
```

BiCGSTAB(L)

```
#include <amgcl/solver/bicgstabl.hpp>
```

```
template <class Backend, class InnerProduct = detail::default_inner_product>
```

```
class amgcl::solver::bicgstabl
```

BiCGStab(L) method. Generalization of BiCGStab method [Sidi93].

Public Functions

```
bicgstabl (size_t n, const params &prm = params (), const backend_params &backend_prm = backend_params(), const InnerProduct &inner_product = InnerProduct())
    Preallocates necessary data structures for the system of size n.
```

```
struct params
```

Solver parameters.

Public Members

```
template<>
int L
    Order of the method.
```

```
template<>
size_t maxiter
    Maximum number of iterations.
```

```
template<>
scalar_type tol
    Target relative residual error.
```

```
template<>
scalar_type abstol
    Target absolute residual error.
```

GMRES

```
#include <amgcl/solver/gmres.hpp>
```

```
template <class Backend, class InnerProduct = detail::default_inner_product>
```

```
class amgcl::solver::gmres
```

Generalized Minimal Residual (GMRES) method. The Generalized Minimal Residual method is an extension of MINRES (which is only applicable to symmetric systems) to unsymmetric systems [Barr94].

Public Functions

```
gmres (size_t n, const params &prm = params (), const backend_params &backend_prm = backend_params(), const InnerProduct &inner_product = InnerProduct())
    Preallocates necessary data structures for the system of size n.
```

```
struct params
```

Solver parameters.

Public Members

```
template<>
unsigned M
    Number of iterations before restart.
```

```
template<>
unsigned maxiter
    Maximum number of iterations.
```

```
template<>
scalar_type tol
    Target relative residual error.
```

```
template<>
scalar_type abstol
    Target absolute residual error.
```

LGMRES

```
#include <amgcl/solver/lgmres.hpp>
```

```
template <class Backend, class InnerProduct = detail::default_inner_product>
```

```
class amgcl::solver::lgmres
```

“Loose” GMRES. The LGMRES algorithm [BaJM05] is designed to avoid some problems in the convergence in restarted GMRES, and often converges in fewer iterations.

Public Functions

lgmres (size_t *n*, const *params* &*prm* = params (), const backend_params &*bprm* = backend_params()), const InnerProduct &*inner_product* = InnerProduct())
 Preallocates necessary data structures for the system of size *n*.

struct **params**

Solver parameters.

Public Members

template<>
 precondition::type **pside**
 Preconditioning kind (left/right).

template<>
 unsigned **M**
 Number of inner GMRES iterations per each outer iteration.

template<>
 unsigned **K**
 Number of vectors to carry between inner GMRES iterations.

According to [BaJM05], good values are in the range of 1...3. However, note that if you want to use the additional vectors to accelerate solving multiple similar problems, larger values may be beneficial.

template<>
 bool **always_reset**
 Reset augmented vectors between solves.

If the solver is used to repeatedly solve similar problems, then keeping the augmented vectors between solves may speed up subsequent solves. This flag, when set, resets the augmented vectors at the beginning of each solve.

template<>
 bool **store_Av**
 Whether LGMRES should store also $A*v$ in addition to vectors v .

template<>
 size_t **maxiter**
 Maximum number of iterations.

template<>
 scalar_type **tol**
 Target relative residual error.

template<>
 scalar_type **abstol**
 Target absolute residual error.

FGMRES

```
#include <amgcl/solver/fgmres.hpp>
```

```
template <class Backend, class InnerProduct = detail::default_inner_product>
```

```
class amgcl::solver::fgmres
```

Flexible GMRES method. Flexible version of the GMRES method [Saad03].

Public Functions

fgmres (size_t *n*, const *params* &*prm* = params (), const backend_params &*bprm* = backend_params(), const InnerProduct &*inner_product* = InnerProduct())
Preallocates necessary data structures for the system of size *n*.

struct **params**

Solver parameters.

Public Members

template<>
unsigned **M**
Number of inner GMRES iterations per each outer iteration.

template<>
unsigned **maxiter**
Maximum number of iterations.

template<>
scalar_type **tol**
Target relative residual error.

template<>
scalar_type **abstol**
Target absolute residual error.

Runtime interface

The compile-time configuration of the solvers used in AMGCL is not always convenient, especially if the solvers are used inside a software package or another library. That is why AMGCL provides runtime interface, which allows to postpone the configuration until, well, runtime. The classes inside `amgcl::runtime` namespace correspond to their compile-time alternatives, but the only template parameter they have is the backend to use.

Since there is no way of knowing the parameter structure at compile time, the runtime classes accept parameters only in form of `boost::property_tree::ptree`. The actual components of the method are set through the parameter tree as well. The runtime interface provides some enumerations for this purpose. For example, to select smoothed aggregation for coarsening, we could do this:

```
boost::property_tree::ptree prm;  
prm.put ("precond.coarsening.type", amgcl::runtime::coarsening::smoothed_aggregation);
```

The enumerations provide functions for converting to/from strings, so the following would work as well:

```
prm.put ("precond.coarsening.type", "smoothed_aggregation");
```

Here is an example of using a runtime-configurable solver:

```
#include <amgcl/backend/builtin.hpp>  
#include <amgcl/runtime.hpp>  
  
...  
  
boost::property_tree::ptree prm;
```

```

prm.put ("precond.coarsening.type", amgcl::runtime::coarsening::smoothed_aggregation);
prm.put ("precond.relaxation.type", amgcl::runtime::relaxation::spai0);
prm.put ("solver.type",             amgcl::runtime::solver::gmres);

amgcl::make_solver<
  amgcl::runtime::amg<Backend>,
  amgcl::runtime::iterative_solver<Backend>
> solve(A, prm);

```

Classes

AMG preconditioner

template <class Backend>

class amgcl::runtime::amg

Runtime-configurable AMG preconditioner.

Inherits from noncopyable

Public Functions

template <class Matrix>

amg (const Matrix &A, params prm = params(), const backend_params &backend_prm = backend_params())

Constructs the AMG hierarchy for the system matrix A. prm is an instance of boost::property_tree::ptree class. The property tree may contain parameters “coarsening.type” and “relax.type”. Default values are amgcl::runtime::coarsening::smoothed_aggregation and runtime::relaxation::spai0. The rest of the property tree should copy the structure of the corresponding amgcl::amg::params struct. For example, when smoothed aggregation is selected for coarsening, one could:

```
prm.put ("coarsening.aggr.eps_strong", 1e-2);
```

Note: Any parameters that are not relevant to the selected AMG components are silently ignored.

template <class Vec1, class Vec2>

void **cycle** (const Vec1 &rhs, Vec2 &x) const

Performs single V-cycle for the given right-hand side rhs and solution x.

template <class Vec1, class Vec2>

void **apply** (const Vec1 &rhs, Vec2 &x) const

Performs single V-cycle for the given right-hand side rhs after clearing x. This is intended for use as a preconditioning procedure.

const matrix &**system_matrix** () const

Returns the system matrix in the backend format

size_t **size** () const

Returns the problem size at the finest level.

Friends

`std::ostream &operator<<` (`std::ostream &os`, `const amg &a`)
Prints some info about the AMG hierarchy to the output stream.

enum `amgcl::runtime::coarsening::type`

Values:

ruge_stuben

Ruge-Stueben coarsening.

aggregation

Aggregation.

smoothed_aggregation

Smoothed aggregation.

smoothed_aggr_emin

Smoothed aggregation with energy minimization.

enum `amgcl::runtime::relaxation::type`

Relaxation schemes.

Values:

gauss_seidel

Gauss-Seidel smoothing.

ilu0

Incomplete LU with zero fill-in.

iluk

Level-based incomplete LU.

ilut

Incomplete LU with thresholding.

damped_jacobi

Damped Jacobi.

spai0

Sparse approximate inverse of 0th order.

spai1

Sparse approximate inverse of 1st order.

chebyshev

Chebyshev relaxation.

Iterative solver

template `<class Backend, class InnerProduct = amgcl::solver::detail::default_inner_product>`

class `amgcl::runtime::iterative_solver`

This is runtime wrapper around AMGCL iterative solver types. Allows to select the actual solver at runtime.

Public Functions

iterative_solver (`size_t n`, `params prm = params()`, `const backend_params &bprm = backend_params()`, `const InnerProduct &inner_product = InnerProduct()`)

Constructs the iterative solver for the problem size `n`. The property tree `solver_prm` may con-

tain “type” entry that would determine the actual type of the iterative solver. Default value: `amgcl::runtime::solver::bicgstab`.

Note: Any parameters that are not relevant to the selected solver are silently ignored.

template <class Matrix, class Precond, class Vec1, class Vec2>

`boost::tuple<size_t, scalar_type> operator ()` (Matrix **const** &A, Precond **const** &P, Vec1 **const** &rhs, Vec2 &&x) **const**

Computes the solution for the given system matrix A and the right-hand side rhs. Returns the number of iterations made and the achieved residual as a `boost::tuple`. The solution vector x provides initial approximation in input and holds the computed solution on output.

The system matrix may differ from the matrix used during initialization. This may be used for the solution of non-stationary problems with slowly changing coefficients. There is a strong chance that a preconditioner built for a time step will act as a reasonably good preconditioner for several subsequent time steps [DeSh12].

template <class Precond, class Vec1, class Vec2>

`boost::tuple<size_t, scalar_type> operator ()` (Precond **const** &P, Vec1 **const** &rhs, Vec2 &&x) **const**

Computes the solution for the given right-hand side rhs. The system matrix is the same that was used for the setup of the preconditioner P. Returns the number of iterations made and the achieved residual as a `boost::tuple`. The solution vector x provides initial approximation in input and holds the computed solution on output.

enum `amgcl::runtime::solver::type`

Values:

cg

Conjugate gradients method.

bicgstab

BiConjugate Gradient Stabilized.

bicgstabl

BiCGStab(ell)

gmres

GMRES.

lgmres

LGMRES.

fgmres

FGMRES.

idrs

IDR(s)

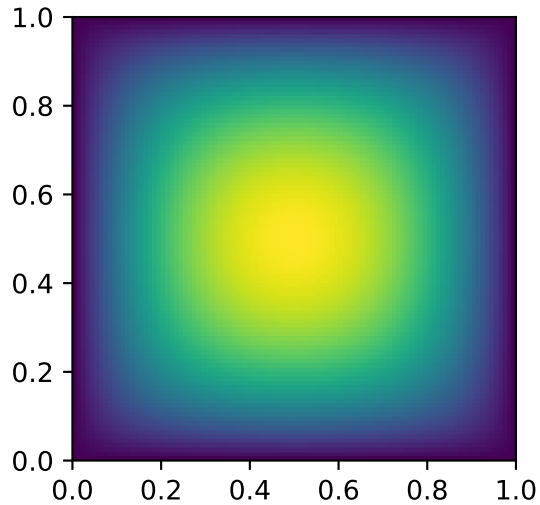
Examples

Assembling matrix for Poisson’s equation

The section provides an example of assembling the system matrix and the right-hand side for a Poisson’s equation in a unit square $\Omega = [0, 1] \times [0, 1]$:

$$-\Delta u = 1, u \in \Omega \quad u = 0, u \in \partial\Omega$$

The solution to the problem looks like this:



Here is how the problem may be discretized on a uniform $n \times n$ grid:

```
#include <vector>

// Assembles matrix for Poisson's equation with homogeneous
// boundary conditions on a n x n grid.
// Returns number of rows in the assembled matrix.
// The matrix is returned in the CRS components ptr, col, and val.
// The right-hand side is returned in rhs.
int poisson(
    int n,
    std::vector<int> &ptr,
    std::vector<int> &col,
    std::vector<double> &val,
    std::vector<double> &rhs
)
{
    int n2 = n * n; // Number of points in the grid.
    double h = 1.0 / (n - 1); // Grid spacing.

    ptr.clear(); ptr.reserve(n2 + 1); ptr.push_back(0);
    col.clear(); col.reserve(n2 * 5); // We use 5-point stencil, so the matrix
    val.clear(); val.reserve(n2 * 5); // will have at most n2 * 5 nonzero elements.

    rhs.resize(n2);

    for(int j = 0, k = 0; j < n; ++j) {
        for(int i = 0; i < n; ++i, ++k) {
            if (i == 0 || i == n - 1 || j == 0 || j == n - 1) {
                // Boundary point. Use Dirichlet condition.
                col.push_back(k);
                val.push_back(1.0);

                rhs[k] = 0.0;
            } else {
                // Interior point. Use 5-point finite difference stencil.
            }
        }
    }
}
```

```

        col.push_back(k - n);
        val.push_back(-1.0 / (h * h));

        col.push_back(k - 1);
        val.push_back(-1.0 / (h * h));

        col.push_back(k);
        val.push_back(4.0 / (h * h));

        col.push_back(k + 1);
        val.push_back(-1.0 / (h * h));

        col.push_back(k + n);
        val.push_back(-1.0 / (h * h));

        rhs[k] = 1.0;
    }

    ptr.push_back(col.size());
}

return n2;
}

```

Adapting custom matrix class

This example shows how to adapt a custom matrix class for use with AMGCL solvers. Let's say the user application declares the following class to store its sparse matrices:

```

class sparse_matrix {
public:
    typedef std::map<int, double> sparse_row;

    sparse_matrix(int n, int m) : _n(n), _m(m), _rows(n) { }

    int nrows() const { return _n; }
    int ncols() const { return _m; }
    int nonzeros() const {
        int nnz = 0;
        for(auto &row : _rows) nnz += row.size();
        return nns;
    }

    // Get a value at row i and column j
    double operator()(int i, int j) const {
        sparse_row::const_iterator elem = _rows[i].find(j);
        return elem == _rows[i].end() ? 0.0 : elem->second;
    }

    // Get reference to a value at row i and column j
    double& operator()(int i, int j) { return _rows[i][j]; }

    // Access the whole row
    const sparse_row& operator[](int i) const { return _rows[i]; }
private:

```

```
int _n, _m;
std::vector<sparse_row> _rows;
};
```

Using `std::map` to store sparse rows is probably not the best idea performance-wise, but it may be convenient during assembly phase. Also, AMGCL will copy the matrix to internal structures during construction, so setup performance should not be affected by the choice of the input matrix type too much.

In order to make the above class work as input matrix for AMGCL, we have to specialize a few templates inside `amgcl::backend` namespace (the templates are declared inside `<amgcl/backend/interface.hpp>`). First we need to let AMGCL know the value type of the matrix:

```
namespace amgcl {
namespace backend {

template <> struct value_type<sparse_matrix> {
    typedef double type;
};
```

We also need to tell how to get the dimensions of the matrix and the number of its nonzero elements:

```
// Number of rows in the matrix
template<> struct rows_impl<sparse_matrix> {
    static int get(const sparse_matrix &A) { return A.nrows(); }
};

// Number of cols in the matrix
template<> struct cols_impl<sparse_matrix> {
    static int get(const sparse_matrix &A) { return A.ncols(); }
};

// Number of nonzeros in the matrix. This may be just a rough estimate.
template<> struct nonzeros_impl<sparse_matrix> {
    static int get(const sparse_matrix &A) { return A.nonzeros(); }
};
```

The last and the most involved part is providing a row iterator for the custom matrix type. In order to do this we need to define a `row_iterator<sparse_matrix>::type` class and specialize `row_begin_impl<sparse_matrix>` template that would return the iterator over the given row of the matrix. Here goes:

```
// Here we define row_iterator<sparse_matrix>::type
template<> struct row_iterator<sparse_matrix> {
    struct iterator {
        sparse_matrix::sparse_row::const_iterator _it, _end;

        // Take the matrix and the row number:
        iterator(const sparse_matrix &A, int row)
            : _it(A[row].begin()), _end(A[row].end()) { }

        // Check if the iterator is valid:
        operator bool() const {
            return _it != _end;
        }

        // Advance to the next nonzero element.
        iterator& operator++() {
            ++_it;
        }
    };
};
```



```

        return *this;
    }

    // Column number of the current nonzero element.
    int col() const { return _it->first; }

    // Value of the current nonzero element.
    double value() const { return _it->second; }
};

typedef iterator type;
};

// Provide a way to obtain the row iterator for the given matrix row:
template<> struct row_begin_impl<sparse_matrix> {
    typedef typename row_iterator<sparse_matrix>::type iterator;
    static iterator get(const sparse_matrix &A, int row) {
        return iterator(A, row);
    }
};

} // namespace backend
} // namespace amgcl

```

After this, we can directly use our matrix type to create an AMGCL solver:

```

// Discretize a 1D Poisson problem
const int n = 10000;

sparse_matrix A(n, n);
for(int i = 0; i < n; ++i) {
    if (i == 0 || i == n - 1) {
        // Dirichlet boundary condition
        A(i,i) = 1.0;
    } else {
        // Internal point.
        A(i, i-1) = -1.0;
        A(i, i) = 2.0;
        A(i, i+1) = -1.0;
    }
}

// Create an AMGCL solver for the problem.
typedef amgcl::backend::builtin<double> Backend;

amgcl::make_solver<
    amgcl::amg<
        Backend,
        amgcl::coarsening::aggregation,
        amgcl::relaxation::spai0
    >,
    amgcl::solver::cg<Backend>
> solve( A );

```

Note: The complete source code of the example may be found at [examples/custom_adapter.cpp](#).

Bibliography

Indices and tables

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