

---

# **allantools Documentation**

*Release 2016.11*

**by the allantools team**

November 27, 2016



<b>1</b>	<b>AllanTools</b>	<b>1</b>
<b>2</b>	<b>Documentation</b>	<b>3</b>
<b>3</b>	<b>IPython notebooks with examples</b>	<b>5</b>
<b>4</b>	<b>Authors</b>	<b>7</b>
<b>5</b>	<b>Installation</b>	<b>9</b>
<b>6</b>	<b>Usage</b>	<b>11</b>
<b>7</b>	<b>Tests</b>	<b>13</b>
<b>8</b>	<b>Getting started</b>	<b>15</b>
8.1	Installation . . . . .	15
8.2	Basic usage . . . . .	15
<b>9</b>	<b>API</b>	<b>17</b>
9.1	The Dataset() class . . . . .	17
9.2	The Plot() class . . . . .	18
9.3	Implemented statistics functions . . . . .	20
9.4	Low-level access to the statistics functions . . . . .	20
<b>10</b>	<b>Function listing</b>	<b>23</b>
10.1	Statistics . . . . .	23
10.2	Noise Generation . . . . .	30
10.3	Utilities . . . . .	30
<b>11</b>	<b>Development</b>	<b>33</b>
11.1	To-do list . . . . .	33
11.2	Notes for Pypi . . . . .	33
<b>12</b>	<b>References</b>	<b>35</b>
12.1	Code . . . . .	35
12.2	Papers . . . . .	35
	<b>Bibliography</b>	<b>37</b>



---

## AllanTools

---

A python library for calculating Allan deviation and related time & frequency statistics. GPL v3+ license.

Developed at <https://github.com/aewallin/allantools> and also available on PyPi at <https://pypi.python.org/pypi/AllanTools> Discussion group at <https://groups.google.com/d/forum/allantools>

Input data should be evenly spaced observations of either fractional frequency, or phase in seconds. Deviations are calculated for given tau values in seconds.

These statistics are currently included:

- `adev()` Allan deviation
- `oadev()` overlapping Allan deviation,
- `mdev()` modified Allan deviation,
- `tdev()` Time deviation
- `hdev()` Hadamard deviation
- `ohdev()` overlapping Hadamard deviation
- `totdev()` total Allan deviation
- `mtie()` Maximum time interval error
- `tierms()` Time interval error RMS
- `mtotdev()` Modified total deviation
- `ttotdev()` Time total deviation
- `htotdev()` Hadamard total deviation
- `theo1()` Thêo1 deviation

Noise generators for creating synthetic datasets are also included:

- violet noise with  $f^2$  PSD
- white noise with  $f^0$  PSD
- pink noise with  $f^{-1}$  PSD
- Brownian or random walk noise with  $f^{-2}$  PSD

see `/tests` for tests that compare `allantools` output to other (e.g. `Stable32`) programs. More test data, benchmarks, `ipython` notebooks, and comparisons to known-good algorithms are welcome!



---

## Documentation

---

See /docs for documentation in sphinx format. On Ubuntu this requires the **python-sphinx** and **python-numpydoc** packages. html/pdf documentation using sphinx can be built locally with:

```
/docs$ make html
/docs$ make latexpdf
```

this generates html documentation in docs/\_build/html and pdf documentation in docs/\_build/latex.

The sphinx documentation is also auto-generated online

- <http://allantools.readthedocs.org>





---

## IPython notebooks with examples

---

See `/examples` for some examples in IPython notebook format.

github formats the notebooks into nice web-pages, for example

- <https://github.com/aewallin/allantools/blob/master/examples/noise-color-demo.ipynb>
- <https://github.com/aewallin/allantools/blob/master/examples/three-cornered-hat-demo.ipynb>

todo: add here a very short guide on how to get started with ipython



**Authors**

---

- Anders E.E. Wallin, anders.e.e.wallin “at” gmail.com
- Danny Price, <https://github.com/telegraphic>
- Cantwell G. Carson, carsonc “at” gmail.com
- Frédéric Meynadier, <https://github.com/fmeynadier>



---

## Installation

---

clone from github, then install with:

```
sudo python setup.py install
```

(see *python setup.py -help install* for install options)

or download from pypi:

```
sudo pip install allantools
```



## Usage

New in 2016.11 : simple top-level API, using dedicated classes for data handling and plotting.

```
import allantools # https://github.com/aewallin/allantools/
import numpy as np

# Compute a deviation using the Dataset class
a = allantools.Dataset(data=np.random.rand(1000))
a.compute("mdev")

# Plot it using the Plot class
b = allantools.Plot()
b.plot(a, errorbars=True, grid=True)
# You can override defaults before "show" if needed
b.ax.set_xlabel("Tau (s)")
b.show()
```

Lower-level access to the algorithms is still possible :

```
import allantools # https://github.com/aewallin/allantools/
rate = 1/float(data_interval_in_s) # data rate in Hz
taus = [1,2,4,8,16] # tau-values in seconds
# fractional frequency data
(taus_used, adev, adeverror, adev_n) = allantools.adev(fract_freqdata, data_type='freq', rate=rate, t
# phase data
(taus_used, adev, adeverror, adev_n) = allantools.adev(phasedata, data_type='phase', rate=rate, taus=

# notes:
# - taus_used may differ from taus, if taus has a non-integer multiples
# of data_interval - adeverror assumes 1/sqrt(adev_n) errors
```





---

## Tests

---

The tests compare the output of allantools to other programs such as Stable32. Tests may be run using `py.test` (<http://pytest.org>). Slow tests are marked 'slow' and tests failing because of a known reason are marked 'fails'. To run all tests:

```
$ py.test
```

To exclude known failing tests:

```
$ py.test -m "not fails" --durations=10
```

To exclude tests that run slowly:

```
$ py.test -m "not slow" --durations=10
```

To exclude both (note option change):

```
$ py.test -k "not (slow or fails)" --durations=10
```

To run the above command without installing the package:

```
$ python setup.py test --addopts "-k 'not (fails or slow)'"
```

Test coverage may be obtained with the (<https://pypi.python.org/pypi/coverage>) module:

```
coverage run --source allantools setup.py test --addopts "-k 'not (fails or slow)'"
coverage report # Reports on standard output
coverage html # Writes annotated source code as html in ./htmlcov/
```

On Ubuntu this requires packages **python-pytest** and **python-coverage**.

Testing on multiple python versions can be done with `tox` (<https://testrun.org/tox>)

```
$ tox
```

Tests run continuously on Travis-CI at <https://travis-ci.org/aewallin/allantools>



---

## Getting started

---

### 8.1 Installation

#### 8.1.1 from Github

To clone and install from github:

```
$ git clone https://github.com/awallin/allantools
$ python setup.py install
```

This installs the latest development version of allantools. Cloning the repository gives you examples, tests, test-data, and iPython notebooks also.

#### 8.1.2 from PyPi

To install the latest released version of allantools from PyPi:

```
$ pip install allantools
```

No examples, tests, or test-data is installed from the PyPi package.

### 8.2 Basic usage

For a general description see the API.

#### 8.2.1 Minimal example, phase data

We can call allantools with only one parameter - an array of phase data. This is suitable for time-interval measurements at 1 Hz, for example from a time-interval-counter measuring the 1PPS output of two clocks.

```
import allantools
import pylab as plt
x = allantools.noise.white(10000)          # Generate some phase data, in seconds.
(taus, adevs, errors, ns) = allantools.oadev(x)
# when only one input parameter is given, phase data in seconds is assumed
# when no rate parameter is given, rate=1.0 is the default
# when no taus parameter is given, taus='octave' is the default
```

## 8.2.2 Frequency data example

Note that allantools assumes nondimensional frequency data input. Normalization, by e.g. dividing all datapoints with the average frequency, is left to the user.

```
import allantools
import pylab as plt
t = numpy.logspace(0, 3, 50)          # tau values from 1 to 1000
y = allantools.noise.white(10000)    # Generate some frequency data
r = 12.3 # sample rate in Hz of the input data
(t2, ad, ade, adn) = allantools.oadev(frequency=y, rate=r, taus=t) # Compute the overlapping ADEV
plt.loglog(t2, ad)                   # Plot the results
plt.show()
```

## 8.2.3 variations on taus parameter

The taus parameter can be given in a number of ways:

```
(t, d, e, n) = allantools.adev(my_phase) # omitted, defaults to taus='octave'
(t, d, e, n) = allantools.adev(my_phase, taus=[]) # empty list, defaults to taus='octave'
(t, d, e, n) = allantools.adev(my_phase, taus='all') # 1, 2, 3, 4, 5, 6, 7, ...
(t, d, e, n) = allantools.adev(my_phase, taus='octave') # 1, 2, 4, 8, 16, ...
(t, d, e, n) = allantools.adev(my_phase, taus='decade') # 1, 2, 4, 10, 20, 40, 100, ...
my_taus=[1,5,15,28]
(t, d, e, n) = allantools.adev(my_phase, taus=my_taus) # python list
(t, d, e, n) = allantools.adev(my_phase, taus=numpy.array(my_taus)) # numpy array
```

Further examples are given in the `examples` directory of the package. For more exciting API musings you can read the API.

## 9.1 The Dataset() class

### New in version 2016.11

This class allows simple data handling.

```
class allantools.Dataset (data=None, rate=1.0, data_type='phase', taus=None)
```

Dataset class for Allantools

#### Example

```
import numpy as np
# Load random data
a = allantools.Dataset(data=np.random.rand(1000))
# compute mdev
a.compute("mdev")
print(a.out["stat"])
```

compute() returns the result of the computation and also stores it in the object's `out` member.

#### Methods

<code>compute</code> (function)	Evaluate the passed function with the supplied data.
<code>set_input</code> (data[, rate, data_type, taus])	Optionnal method if you chose not to set inputs on init

```
__init__ (data=None, rate=1.0, data_type='phase', taus=None)
```

Initialize object with input data

#### Parameters data: np.array

Input data. Provide either phase or frequency (fractional, adimensional)

#### rate: float

The sampling rate for data, in Hz. Defaults to 1.0

#### data\_typ: {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to "phase".

#### taus: np.array

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic calculation of taus list

**Returns** Dataset()

A Dataset() instance

**inp** = {'taus': None, 'rate': None, 'data': None, 'data\_type': None}

input data Dict, will be initialized by `__init__()`

**out** = {'stat': None, 'stat\_unc': None, 'stat\_err': None, 'stat\_id': None, 'stat\_n': None, 'taus': None}

output data Dict, to be populated by `compute()`

**set\_input** (*data*, *rate=1.0*, *data\_type='phase'*, *taus=None*)

Optionnal method if you chose not to set inputs on init

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional)

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_typ: {'phase', 'freq'}**

Data type, i.e. phase or frequency. Defaults to "phase".

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic

**compute** (*function*)

Evaluate the passed function with the supplied data.

Stores result in `self.out`.

**Parameters function: str**

Name of the `allantools` function to evaluate

**Returns** result: dict

The results of the calculation.

## 9.2 The Plot() class

**New in version 2016.11**

This class allows simple data plotting.

**class** `allantools.Plot` (*no\_display=False*)

A class for plotting data once computed by Allantools

**Example** `import allantools import numpy as np a = allantools.Dataset(data=np.random.rand(1000))  
a.compute("mdev") b = allantools.Plot() b.plot(a) b.show()`

Uses matplotlib. `self.fig` and `self.ax` stores the return values of `matplotlib.pyplot.subplots()`. `plot()` sets various defaults, but you can change them by using standard matplotlib method on `self.fig` and `self.ax`

## Methods

---

<code>plot(atDataset[, errorbars, grid])</code>	use matplotlib methods for plotting
<code>show()</code>	Calls matplotlib.pyplot.show()

---

`__init__` (*no\_display=False*)

set `no_display` to `True` when we don't have an X-window (e.g. for tests)

`plot` (*atDataset, errorbars=False, grid=False*)

use matplotlib methods for plotting

**Parameters** `atDataset` : `allantools.Dataset()`

a dataset with computed data

**errorbars** : boolean

Plot errorbars. Defaults to `False`

**grid** : boolean

Plot grid. Defaults to `False`

`show` ()

Calls `matplotlib.pyplot.show()`

Keeping this separated from `plot` () allows to tweak display before rendering

## 9.3 Implemented statistics functions

Function	Description
<code>adev()</code>	Allan deviation
<code>oadev()</code>	Overlapping Allan deviation
<code>mdev()</code>	Modified Allan deviation
<code>tdev()</code>	Time deviation
<code>hdev()</code>	Hadamard deviation
<code>ohdev()</code>	Overlapping Hadamard deviation
<code>totdev()</code>	Total deviation
<code>mtotdev()</code>	Modified total deviation
<code>ttotdev()</code>	Time total deviation
<code>htotdev()</code>	Hadamard total deviation
<code>theo1()</code>	Theo1 deviation
<code>mtie()</code>	Maximum Time Interval Error
<code>tierms()</code>	Time Interval Error RMS
<code>gradev()</code>	Gap resistant overlapping Allan deviation

## 9.4 Low-level access to the statistics functions

The deviation functions are generally of the form:

```
(tau_out, adev, adeverr, n) = allantools.adev(data, rate=1.0, data_type="phase", taus=None)
```

*Inputs:*

- **data** = list of phase measurements in seconds, or list of fractional frequency measurements (nondimensional)



- **rate** = sample rate of data in Hz , i.e. interval between phase measurements is 1/rate seconds.
- **data\_type**= = either “phase” or “freq”
- **taus** = list of tau-values for ADEV computation. The keywords “all”, “octave”, or “decade” can also be used.

*Outputs*

- **tau\_out** = list of tau-values for which deviations were computed
- **adev** = list of ADEV (or another statistic) deviations
- **adeverr** = list of estimated errors of allan deviations. some functions instead return a confidence interval (**err\_l**, **err\_h**)
- **n** = list of number of pairs in allan computation. standard error is  $adeverr = adev/\sqrt{n}$



---

## Function listing

---

### 10.1 Statistics

`allantools.adev` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

**Allan deviation.** Classic - use only if required - relatively poor confidence.

$$\sigma_{ADEV}^2(\tau) = \frac{1}{2\tau^2} \langle (x_{n+2} - 2x_{n+1} + x_n)^2 \rangle = \frac{1}{2(N-2)\tau^2} \sum_{n=1}^{N-2} (x_{n+2} - 2x_{n+1} + x_n)^2$$

where  $x_n$  is the time-series of phase observations, spaced by the measurement interval  $\tau$ , and with length  $N$ .

Or alternatively calculated from a time-series of fractional frequency:

$$\sigma_{ADEV}^2(\tau) = \frac{1}{2} \langle (\bar{y}_{n+1} - \bar{y}_n)^2 \rangle$$

where  $\bar{y}_n$  is the time-series of fractional frequency at averaging time  $\tau$

NIST SP 1065 eqn (6) and (7), pages 14 and 15

**Parameters data:** `np.array`

Input data. Provide either phase or frequency (fractional, adimensional).

**rate:** `float`

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type:** {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to "phase".

**taus:** `np.array`

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

**Returns** (`taus2`, `ad`, `ade`, `ns`): tuple

Tuple of values

`taus2`: `np.array`

Tau values for which td computed

`ad`: `np.array`

Computed adev for each tau value

ade: np.array

adev errors

ns: np.array

Values of N used in each adev calculation

`allantools.oadev(data, rate=1.0, data_type='phase', taus=None)`

**overlapping Allan deviation.** General purpose - most widely used - first choice

$$\sigma_{OADEV}^2(m\tau_0) = \frac{1}{2(m\tau_0)^2(N-2m)} \sum_{n=1}^{N-2m} (x_{n+2m} - 2x_{n+m} + x_n)^2$$

where  $\sigma_x^2(m\tau_0)$  is the overlapping Allan deviation at an averaging time of  $\tau = m\tau_0$ , and  $x_n$  is the time-series of phase observations, spaced by the measurement interval  $\tau_0$ , with length  $N$ .

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional).

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type: {'phase', 'freq'}**

Data type, i.e. phase or frequency. Defaults to "phase".

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

**Returns (taus2, ad, ade, ns): tuple**

Tuple of values

taus2: np.array

Tau values for which td computed

ad: np.array

Computed oadev for each tau value

ade: np.array

oadev errors

ns: np.array

Values of N used in each oadev calculation

`allantools.mdev(data, rate=1.0, data_type='phase', taus=None)`

**Modified Allan deviation.** Used to distinguish between White and Flicker Phase Modulation.

$$\sigma_{MDEV}^2(m\tau_0) = \frac{1}{2(m\tau_0)^2(N-3m+1)} \sum_{j=1}^{N-3m+1} \left\{ \sum_{i=j}^{j+m-1} x_{i+2m} - 2x_{i+m} + x_i \right\}^2$$

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional).

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type: {'phase', 'freq'}**

Data type, i.e. phase or frequency. Defaults to "phase".

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

**Returns** (taus2, md, mde, ns): tuple

Tuple of values

taus2: np.array

Tau values for which td computed

md: np.array

Computed mdev for each tau value

mde: np.array

mdev errors

ns: np.array

Values of N used in each mdev calculation

**Notes**

see [http://www.leapsecond.com/tools/adev\\_lib.c](http://www.leapsecond.com/tools/adev_lib.c)

NIST SP 1065 eqn (14) and (15), page 17

`allantools.hdev(data, rate=1.0, data_type='phase', taus=None)`

**Hadamard deviation.** Rejects frequency drift, and handles divergent noise.

$$\sigma_{HDEV}^2(\tau) = \frac{1}{6\tau^2(N-3)} \sum_{i=1}^{N-3} (x_{i+3} - 3x_{i+2} + 3x_{i+1} + x_i)^2$$

where  $x_i$  is the time-series of phase observations, spaced by the measurement interval  $\tau$ , and with length  $N$ .

NIST SP 1065 eqn (17) and (18), page 20

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional).

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type: {'phase', 'freq'}**

Data type, i.e. phase or frequency. Defaults to "phase".

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

`allantools.ohdev` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

**Overlapping Hadamard deviation.** Better confidence than normal Hadamard.

$$\sigma_{OHDEV}^2(m\tau_0) = \frac{1}{6(m\tau_0)^2(N-3m)} \sum_{i=1}^{N-3m} (x_{i+3m} - 3x_{i+2m} + 3x_{i+m} + x_i)^2$$

where  $x_i$  is the time-series of phase observations, spaced by the measurement interval  $\tau_0$ , and with length  $N$ .

**Parameters data:** `np.array`

Input data. Provide either phase or frequency (fractional, adimensional).

**rate:** `float`

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type:** {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to "phase".

**taus:** `np.array`

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

**Returns** (taus2, hd, hde, ns): tuple

Tuple of values

taus2: `np.array`

Tau values for which td computed

hd: `np.array`

Computed hdev for each tau value

hde: `np.array`

hdev errors

ns: `np.array`

Values of N used in each hdev calculation

`allantools.tdev` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

**Time deviation.** Based on modified Allan variance.

$$\sigma_{TDEV}^2(\tau) = \frac{\tau^2}{3} \sigma_{MDEV}^2(\tau)$$

Note that TDEV has a unit of seconds.

**Parameters data:** `np.array`

Input data. Provide either phase or frequency (fractional, adimensional).

**rate:** `float`

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type:** {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to “phase”.

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

**Returns** (taus, tdev, tdev\_error, ns): tuple

Tuple of values

taus: np.array

Tau values for which td computed

tdev: np.array

Computed time deviations (in seconds) for each tau value

tdev\_errors: np.array

Time deviation errors

ns: np.array

Values of N used in `mdev_phase()`

**Notes**

[http://en.wikipedia.org/wiki/Time\\_deviation](http://en.wikipedia.org/wiki/Time_deviation)

`allantools.totdev(data, rate=1.0, data_type='phase', taus=None)`

**Total deviation.** Better confidence at long averages for Allan.

$$\sigma_{TOTDEV}^2(m\tau_0) = \frac{1}{2(m\tau_0)^2(N-2)} \sum_{i=2}^{N-1} (x_{i-m}^* - 2x_i^* + x_{i+m}^*)^2$$

Where  $x_i^*$  is a new time-series of length  $3N - 4$  derived from the original phase time-series  $x_n$  of length  $N$  by reflection at both ends.

FIXME: better description of reflection operation. the original data  $x$  is in the center of  $x^*$ :  $x^*(1-j) = 2x(1) - x(1+j)$  for  $j=1..N-2$   $x^*(i) = x(i)$  for  $i=1..N$   $x^*(N+j) = 2x(N) - x(N-j)$  for  $j=1..N-2$   $x^*$  has length  $3N-4$   $\tau = m \cdot \tau_0$

FIXME: bias correction <http://www.wiley.com/CI2.pdf> page 5

**Parameters phase: np.array**

Phase data in seconds. Provide either phase or frequency.

**frequency: np.array**

Fractional frequency data (nondimensional). Provide either frequency or phase.

**rate: float**

The sampling rate for phase or frequency, in Hz

**taus: np.array**

Array of tau values for which to compute measurement

## References

David A. Howe, *The total deviation approach to long-term characterization of frequency stability*, IEEE tr. UFFC vol 47 no 5 (2000)

NIST SP 1065 eqn (25) page 23

`allantools.mtotdev` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

**PRELIMINARY - REQUIRES FURTHER TESTING.** Modified Total deviation. Better confidence at long averages for modified Allan

FIXME: bias-correction <http://www.wiley.com/CI2.pdf> page 6

The variance is scaled up (divided by this number) based on the noise-type identified. WPM 0.94 FPM 0.83 WFM 0.73 FFM 0.70 RWFM 0.69

### Parameters *data*: np.array

Input data. Provide either phase or frequency (fractional, adimensional).

### *rate*: float

The sampling rate for data, in Hz. Defaults to 1.0

### *data\_type*: {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to "phase".

### *taus*: np.array

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

### NIST SP 1065 eqn (27) page 25

`allantools.ttotdev` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

Time Total Deviation modified total variance scaled by  $\tau^2 / 3$  NIST SP 1065 eqn (28) page 26 <— formula should have tau squared !?!

`allantools.htotdev` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

**PRELIMINARY - REQUIRES FURTHER TESTING.** Hadamard Total deviation. Better confidence at long averages for Hadamard deviation

FIXME: bias corrections from <http://www.wiley.com/CI2.pdf> W FM 0.995 alpha=0 F FM 0.851 alpha=-1 RW FM 0.771 alpha=-2 FW FM 0.717 alpha=-3 RR FM 0.679 alpha=-4

### Parameters *data*: np.array

Input data. Provide either phase or frequency (fractional, adimensional).

### *rate*: float

The sampling rate for data, in Hz. Defaults to 1.0

### *data\_type*: {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to "phase".

### *taus*: np.array

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.



`allantools.theo1` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

**PRELIMINARY - REQUIRES FURTHER TESTING.** Theo1 is a two-sample variance with improved confidence and extended averaging factor range.

$$\sigma_{THEO1}^2(m\tau_0) = \frac{1}{(m\tau_0)^2(N-m)} \sum_{i=1}^{N-m} \sum_{\delta=0}^{m/2-1} \frac{1}{m/2-\delta} \{(x_i - x_{i-\delta+m/2}) + (x_{i+m} - x_{i+\delta+m/2})\}^2$$

Where  $10 \leq m \leq N - 1$  is even.

FIXME: bias correction

NIST SP 1065 eq (30) page 29

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional).

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type: {'phase', 'freq'}**

Data type, i.e. phase or frequency. Defaults to "phase".

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

`allantools.mtie` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

Maximum Time Interval Error.

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional).

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type: {'phase', 'freq'}**

Data type, i.e. phase or frequency. Defaults to "phase".

**taus: np.array**

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

## Notes

this seems to correspond to Stable32 setting "Fast(u)" Stable32 also has "Decade" and "Octave" modes where the dataset is extended somehow?

`allantools.tierms` (*data*, *rate*=1.0, *data\_type*='phase', *taus*=None)

Time Interval Error RMS.

**Parameters data: np.array**

Input data. Provide either phase or frequency (fractional, adimensional).

**rate: float**

The sampling rate for data, in Hz. Defaults to 1.0

**data\_type:** {'phase', 'freq'}

Data type, i.e. phase or frequency. Defaults to "phase".

**taus:** np.array

Array of tau values, in seconds, for which to compute statistic. Optionally set `taus=["all"|"octave"|"decade"]` for automatic tau-list generation.

## 10.2 Noise Generation

`allantools.noise.white` (*num\_points=1024, b0=1.0, fs=1.0*)

generate time series with white noise that has constant PSD =  $b_0$ , up to the nyquist frequency  $fs/2$   $N$  = number of samples  $b_0$  = desired power-spectral density in  $[X^2/Hz]$  where  $X$  is the unit of  $x$   $fs$  = sampling frequency, i.e.  $1/fs$  is the time-interval between datapoints

the pre-factor corresponds to the area 'box' under the PSD-curve: The PSD is at 'height'  $b_0$  and extends from 0 Hz up to the nyquist frequency  $fs/2$

`allantools.noise.brown` (*num\_points=1024, b2=1.0, fs=1.0*)

Brownian or random walk (diffusion) noise with  $1/f^2$  PSD (not really a color... rather Brownian or random-walk)

$N$  = number of samples  $b_2$  = desired PSD is  $b_2 * f^{-2}$   $fs$  = sampling frequency

we integrate white-noise to get Brownian noise.

`allantools.noise.violet` (*num\_points*)

violet noise with  $f^2$  PSD

`allantools.noise.pink` (*N, depth=80*)

$N$ -length vector with (approximate) pink noise pink noise has  $1/f$  PSD

## 10.3 Utilities

`allantools.frequency2phase` (*freqdata, rate*)

integrate fractional frequency data and output phase data

**Parameters** `freqdata:` np.array

Data array of fractional frequency measurements (nondimensional)

**rate:** float

The sampling rate for phase or frequency, in Hz

**Returns** `phasedata:` np.array

Time integral of fractional frequency data, i.e. phase (time) data in units of seconds. For phase in units of radians, see `phase2radians()`

`allantools.phase2frequency` (*phase, rate*)

Convert phase in seconds to fractional frequency

**Parameters** `phase:` np.array

Data array of phase in seconds

**rate:** float

The sampling rate for phase, in Hz

**Returns** *y*:

Data array of fractional frequency

`allantools.phase2radians` (*phasedata*, *v0*)

Convert phase in seconds to phase in radians

**Parameters** *phasedata*: **np.array**

Data array of phase in seconds

**v0**: **float**

Nominal oscillator frequency in Hz

**Returns** *fi*:

phase data in radians

`allantools.edf_simple` (*N*, *m*, *alpha*)

Equivalent degrees of freedom. Simple approximate formulae.

**Parameters** *N*: **int**

the number of phase samples

**m**: **int**

averaging factor,  $\tau = m * \tau_0$

**alpha**: **int**

exponent of *f* for the frequency PSD: 'wp' returns white phase noise.  $\alpha=+2$  'wf' returns white frequency noise.  $\alpha=0$  'fp' returns flicker phase noise.  $\alpha=+1$  'ff' returns flicker frequency noise.  $\alpha=-1$  'rf' returns random walk frequency noise.  $\alpha=-2$  If the input is not recognized, it defaults to idealized, uncorrelated noise with (*N*-1) degrees of freedom.

**Returns** *edf*: **float**

Equivalent degrees of freedom

## Notes

S. Stein, Frequency and Time - Their Measurement and Characterization. Precision Frequency Control Vol 2, 1985, pp 191-416. <http://tf.boulder.nist.gov/general/pdf/666.pdf>

`allantools.edf_greenhall` (*alpha*, *d*, *m*, *N*, *overlapping=False*, *modified=False*, *verbose=True*)

Used for the following deviations (see <http://www.wiley.com/CI2.pdf> page 8) *adev()* *oadev()* *mdev()* *tdev()* *hdev()* *ohdev()*

`allantools.edf_totdev` (*N*, *m*, *alpha*)

Equivalent degrees of freedom for Total Deviation

`allantools.edf_mtdev` (*N*, *m*, *alpha*)

Equivalent degrees of freedom for Modified Total Deviation

`allantools.three_cornered_hat_phase` (*phasedata\_ab*, *phasedata\_bc*, *phasedata\_ca*, *rate*, *taus*, *function*)

Three Cornered Hat Method

Given three clocks A, B, C, we seek to find their variances  $\sigma_A^2$ ,  $\sigma_B^2$ ,  $\sigma_C^2$ . We measure three phase differences, assuming no correlation between the clocks, the measurements have variances:

$$\begin{aligned}\sigma_{AB}^2 &= \sigma_A^2 + \sigma_B^2 \\ \sigma_{BC}^2 &= \sigma_B^2 + \sigma_C^2 \\ \sigma_{CA}^2 &= \sigma_C^2 + \sigma_A^2\end{aligned}$$

Which allows solving for the variance of one clock as:

$$\sigma_A^2 = \frac{1}{2}(\sigma_{AB}^2 + \sigma_{CA}^2 - \sigma_{BC}^2)$$

and similarly cyclic permutations for  $\sigma_B^2$  and  $\sigma_C^2$

**Parameters phasedata\_ab: np.array**

phase measurements between clock A and B, in seconds

**phasedata\_bc: np.array**

phase measurements between clock B and C, in seconds

**phasedata\_ca: np.array**

phase measurements between clock C and A, in seconds

**rate: float**

The sampling rate for phase, in Hz

**taus: np.array**

The tau values for deviations, in seconds

**function: allantools deviation function**

The type of statistic to compute, e.g. allantools.oadev

**Returns tau\_ab: np.array**

Tau values corresponding to output deviations

**dev\_a: np.array**

List of computed values for clock A

## References

<http://www.wiley.com/3-CornHat.htm>

## 11.1 To-do list

Here follows an un-ordered to do list:

- Statistics and core algorithms
  - The `mtie_phase_fast` approach to MTIE, using a binary tree (see BREGNI reference)
  - TheoH
  - Confidence intervals based on identified noise-type and equivalent degrees of freedom.
  - Bias corrections for biased statistics (`totdev`, `mtotdev`, `htotdev`, `theo1`)
  - Multi-threading for faster processing of (very) large datasets
  - Faster algorithms for `mtotdev()` and `htotdev()` which are currently very slow
- Improve documentation
- Improve packaging for PyPi and/or other packaging systems (PPA for Ubuntu/Debian?)
- Stable32-style plots using matplotlib
- Tests for different noise types according to IEEE 1139, include power-spectral-density calculations
- Conversion between phase noise and Allan variance
- Phase noise calculations and plots
- Comparison to other libraries such as GPSTk

Make sure your patch does not break any of the tests, and does not significantly reduce the readability of the code.

## 11.2 Notes for Pypi

Creating a source distribution

```
python setup.py sdist
```

Testing the source distribution. The install takes a long time while compiling numpy and scipy.

```
$ virtualenv tmp
$ tmp/bin/pip install dist/AllanTools-2016.2.tar.gz
$ tmp/bin/python
>>> import allantools
```

Registering, uploading and testing source distribution to PyPi test server (requires a ~/.pypirc with username and password)

```
$ python setup.py register -r test
$ python setup.py sdist upload -r test
$ pip install -i https://testpypi.python.org/pypi AllanTools
```

Registering and uploading to PyPi

```
$ python setup.py register
$ python setup.py sdist upload
```

## References

---

### 12.1 Code

- <http://www.mathworks.com/matlabcentral/fileexchange/26659-allan-v3-0>
- <http://www.mathworks.com/matlabcentral/fileexchange/26637-allanmodified>
- [http://www.leapsecond.com/tools/adev\\_lib.c](http://www.leapsecond.com/tools/adev_lib.c)

### 12.2 Papers





- [Greenhall2004] Greenhall & Riley, “UNCERTAINTY OF STABILITY VARIANCES BASED ON FINITE DIFFERENCES” 35th Annual Precise Time and Time Interval (PTTI) Meeting <https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20050061319.pdf>
- [wikipedia\_adev] [http://en.wikipedia.org/wiki/Allan\\_variance](http://en.wikipedia.org/wiki/Allan_variance)
- [SP1065] NIST Special Publication 1065 <http://tf.nist.gov/timefreq/general/pdf/2220.pdf>
- [IEEE1139] 1139-2008 - IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology - Random Instabilities <http://dx.doi.org/10.1109/IEEESTD.2008.4797525>
- [Riley\_Stable32] <http://www.stable32.com/Handbook.pdf>
- [Riley\_CI] Confidence Intervals and Bias Corrections for the Stable32 Variance Functions W.J. Riley, Hamilton Technical Services <http://www.wriley.com/CI2.pdf>
- [Vernotte2011] F. Vernotte, “Variance Measurements”, 2011 IFCS & EFTF [http://www.ieee-uffc.org/frequency-control/learning/pdf/Vernotte-Variance\\_Measurements.pdf](http://www.ieee-uffc.org/frequency-control/learning/pdf/Vernotte-Variance_Measurements.pdf)
- [Stein1985] S. Stein, Frequency and Time - Their Measurement and Characterization. Precision Frequency Control Vol 2, 1985, pp 191-416. <http://tf.boulder.nist.gov/general/pdf/666.pdf>
- [Riley\_1] W.J.Riley, “THE CALCULATION OF TIME DOMAIN FREQUENCY STABILITY” <http://www.wriley.com/paper1ht.htm>
- [Sesia2011] SESIA I., GALLEANI L., TAVELLA P., Application of the Dynamic Allan Variance for the Characterization of Space Clock Behavior, <http://dx.doi.org/10.1109/TAES.2011.5751232>
- [Bregni2001] S. BREGNI, Fast Algorithms for TVAR and MTIE Computation in Characterization of Network Synchronization Performance. [http://home.deib.polimi.it/bregni/papers/csc2001\\_fastalgo.pdf](http://home.deib.polimi.it/bregni/papers/csc2001_fastalgo.pdf)
- [Howe2000] David A. Howe, The total deviation approach to long-term characterization of frequency stability, IEEE tr. UFFC vol 47 no 5 (2000) <http://dx.doi.org/10.1109/58.869040>
- [Sesia2008] Ilaria Sesia and Patrizia Tavella, Estimating the Allan variance in the presence of long periods of missing data and outliers. 2008 Metrologia 45 S134 <http://dx.doi.org/10.1088/0026-1394/45/6/S19>
- [Howe\_theo1] D.A. Howe and T.N. Tasset THEO1: CHARACTERIZATION OF VERY LONG-TERM FREQUENCY STABILITY <http://tf.nist.gov/general/pdf/1990.pdf>



## Symbols

`__init__()` (allantools.Dataset method), 17  
`__init__()` (allantools.Plot method), 20

## A

`adev()` (in module allantools), 23

## B

`brown()` (in module allantools.noise), 30

## C

`compute()` (allantools.Dataset method), 18

## D

Dataset (class in allantools), 17

## E

`edf_greenhall()` (in module allantools), 31  
`edf_mtdev()` (in module allantools), 31  
`edf_simple()` (in module allantools), 31  
`edf_totdev()` (in module allantools), 31

## F

`frequency2phase()` (in module allantools), 30

## H

`hdev()` (in module allantools), 25  
`htotdev()` (in module allantools), 28

## I

`inp` (allantools.Dataset attribute), 18

## M

`mdev()` (in module allantools), 24  
`mtie()` (in module allantools), 29  
`mtotdev()` (in module allantools), 28

## O

`oadev()` (in module allantools), 24

`ohdev()` (in module allantools), 26  
`out` (allantools.Dataset attribute), 18

## P

`phase2frequency()` (in module allantools), 30  
`phase2radians()` (in module allantools), 31  
`pink()` (in module allantools.noise), 30  
Plot (class in allantools), 18  
`plot()` (allantools.Plot method), 20

## S

`set_input()` (allantools.Dataset method), 18  
`show()` (allantools.Plot method), 20

## T

`tdev()` (in module allantools), 26  
`theo1()` (in module allantools), 28  
`three_cornered_hat_phase()` (in module allantools), 31  
`tierms()` (in module allantools), 29  
`totdev()` (in module allantools), 27  
`ttotdev()` (in module allantools), 28

## V

`violet()` (in module allantools.noise), 30

## W

`white()` (in module allantools.noise), 30